Coastal Sediment Transport and morphology

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- Sediment transport by waves and current
- Longshore sediment transport
- Coastline changes
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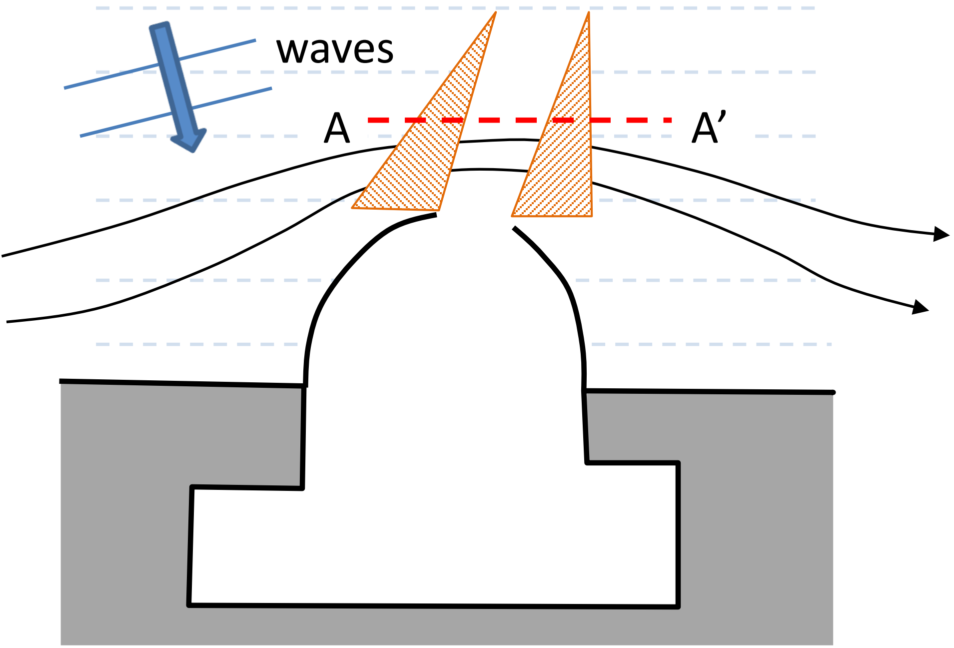
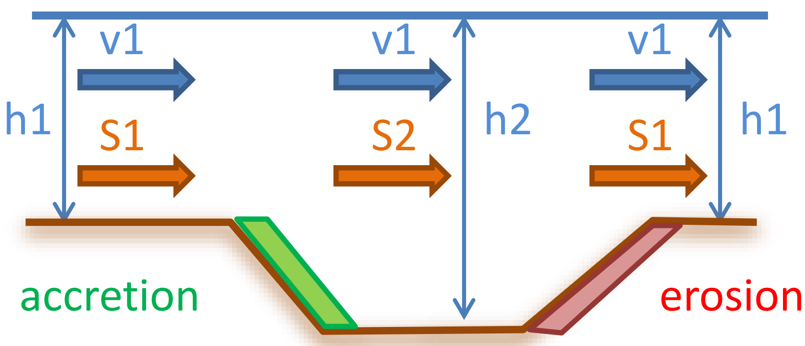
- Dune erosion

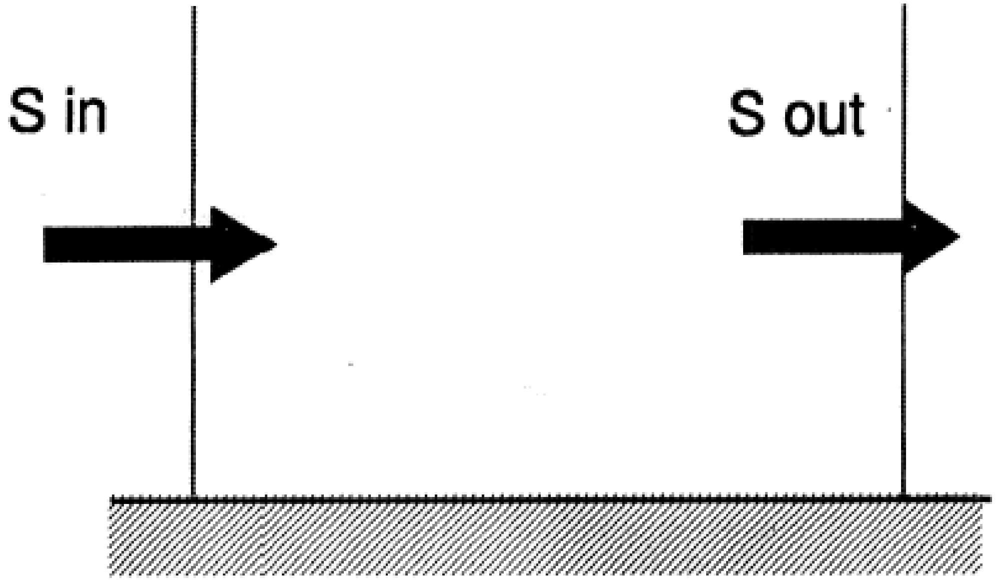
1) Siltation navigation channel



Section A – A'



h2 > h1 → v2 < v1
S2 < S1 (siltation)

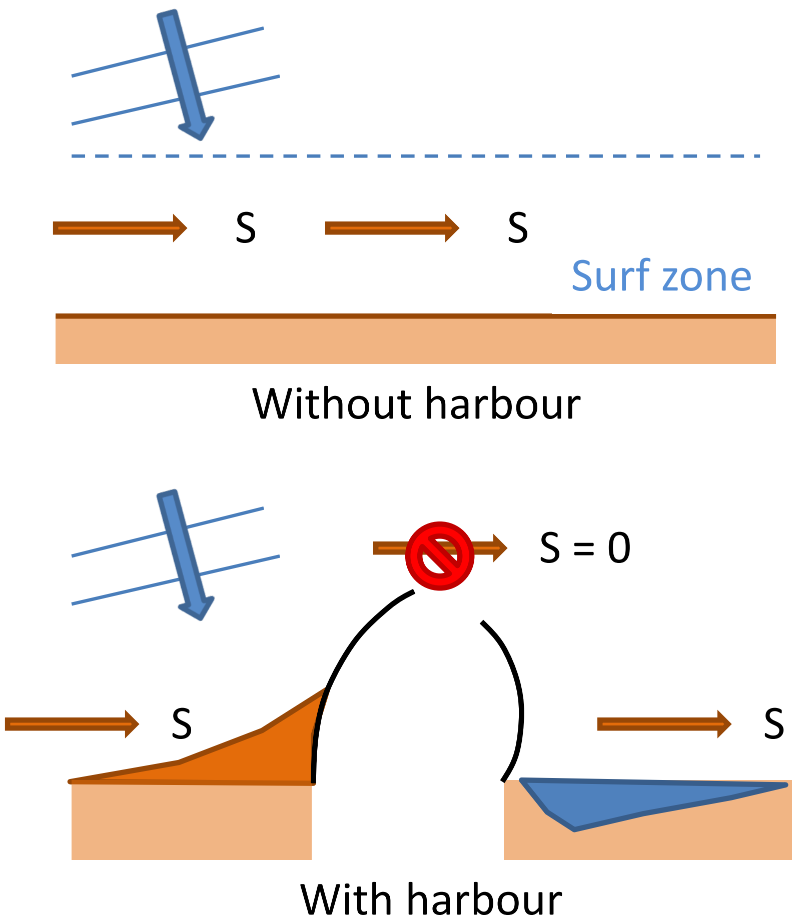


S in = S out → stable

S in > S out → accretion

S in < S out → erosion

2) Accretion / erosion near harbour

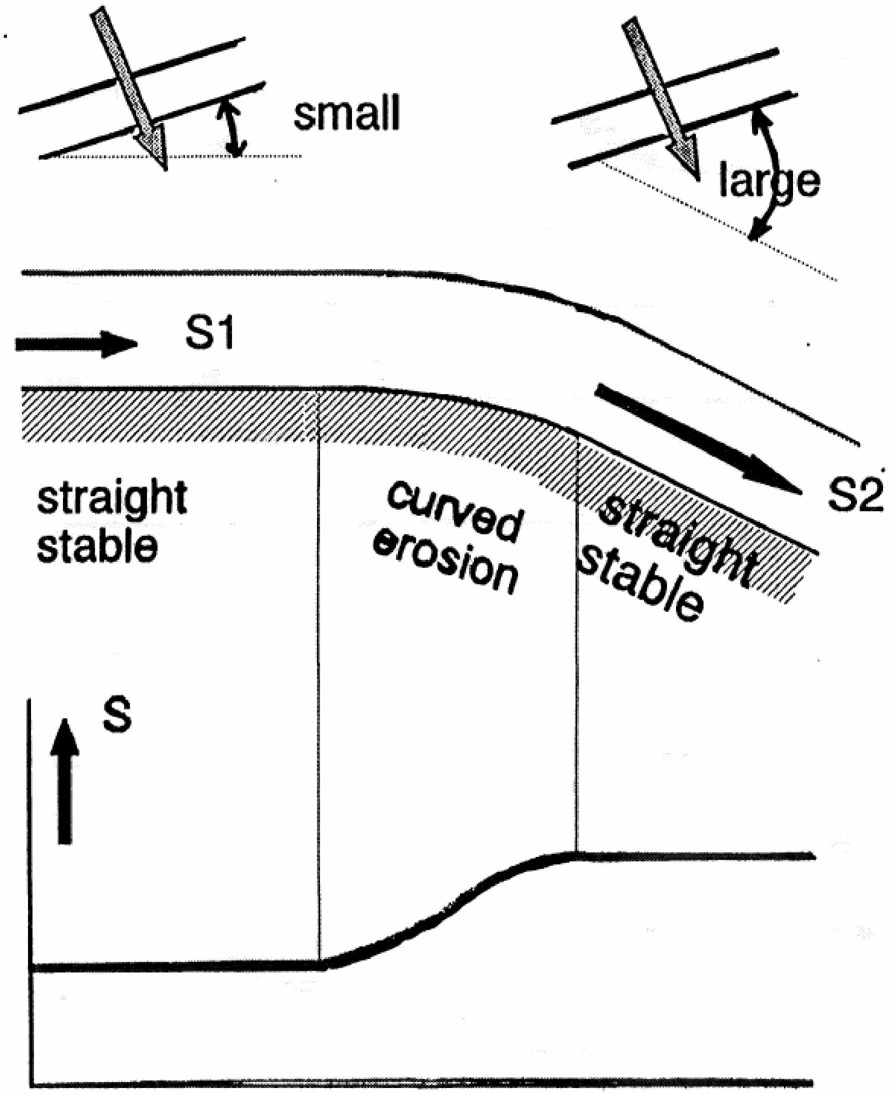


Per year: S m^3

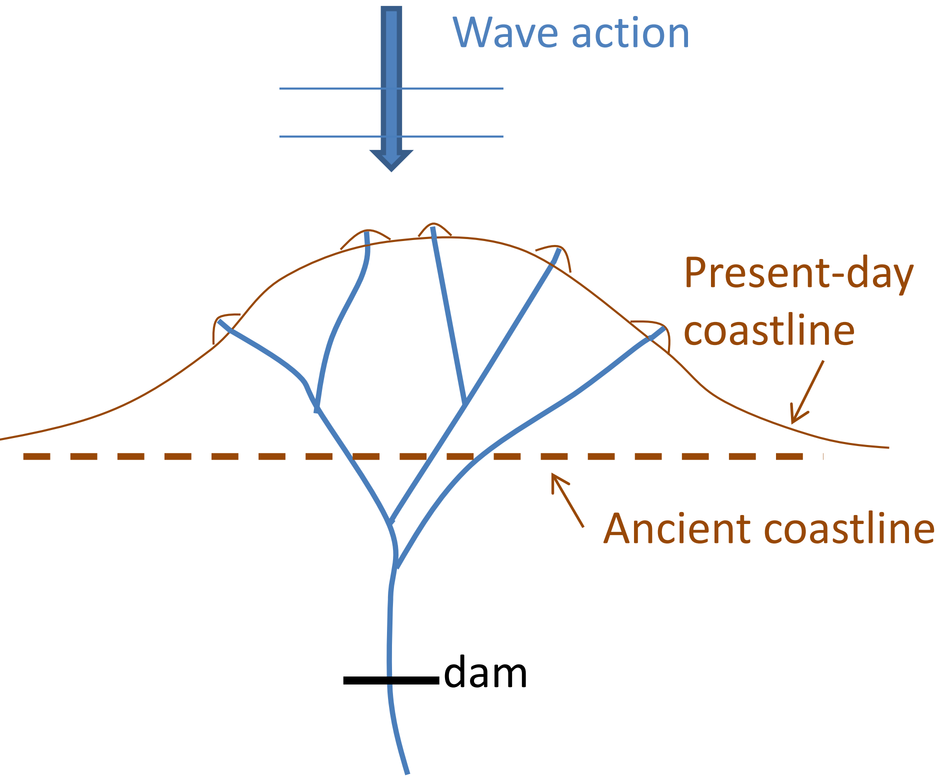
Accretion after n years: n S m^3

Erosion after n years: n S m^3

3) Curved coastline



Distance alongshore



- Dam blocks sediment supply to the delta
- Delta lobes are eroded by wave action

- Longshore transport away from the delta mouth, depending on wave climate
- Diffusional process

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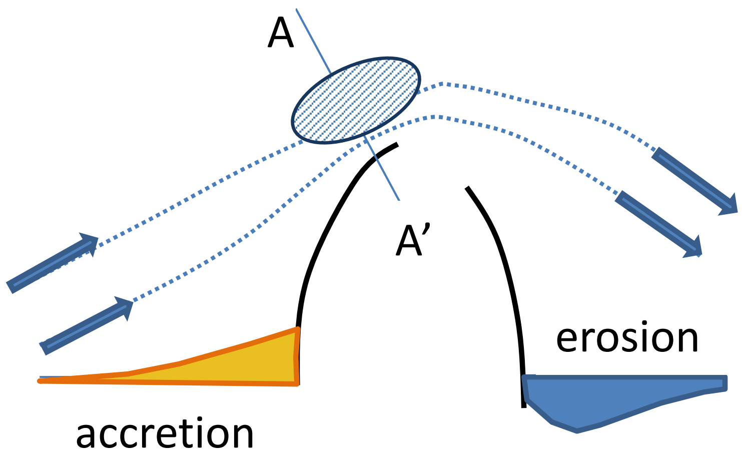
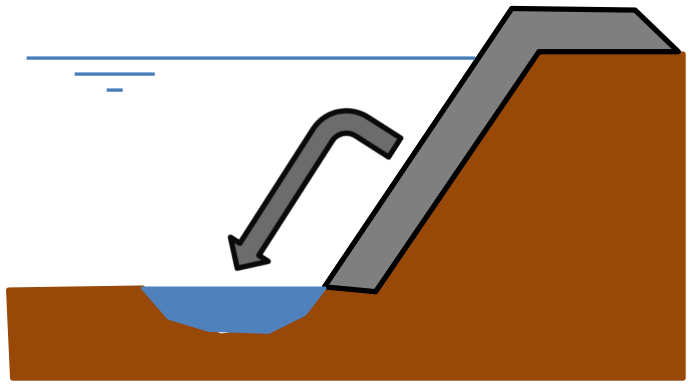
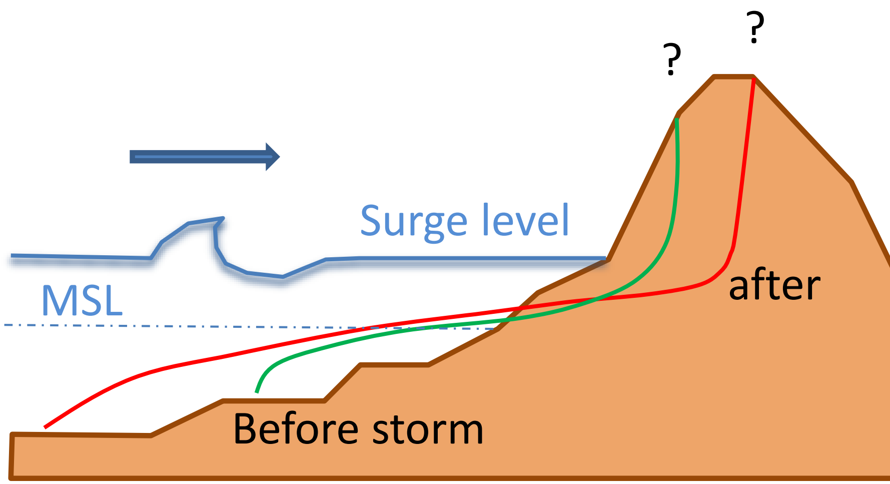
5) Erosion near a breakwater



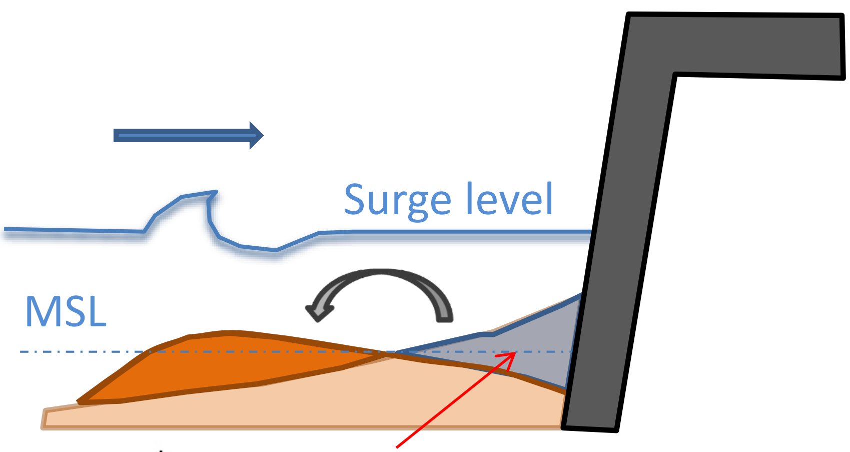


Section A – A’

7) Dune erosion / erosion pit



Red or green is the question



Magnitude of erosion

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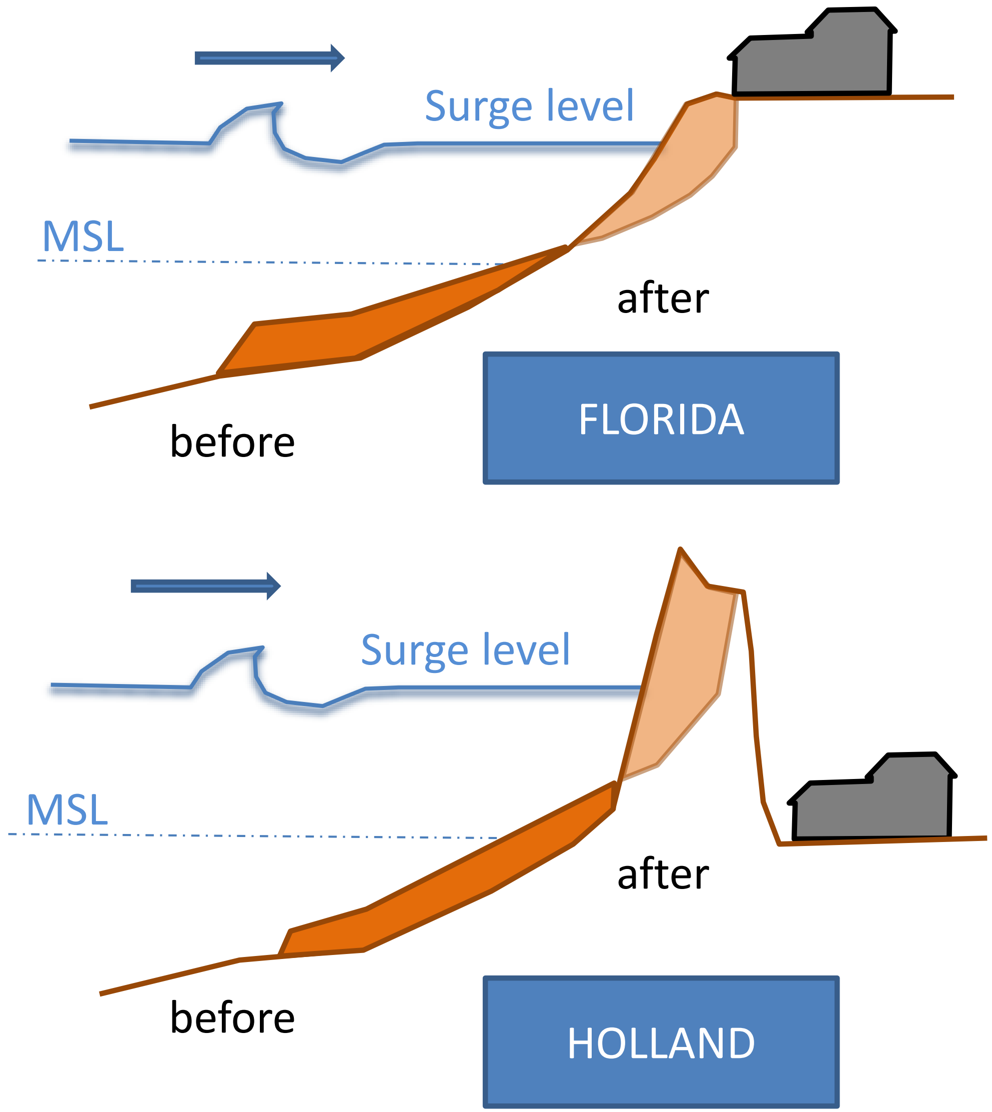
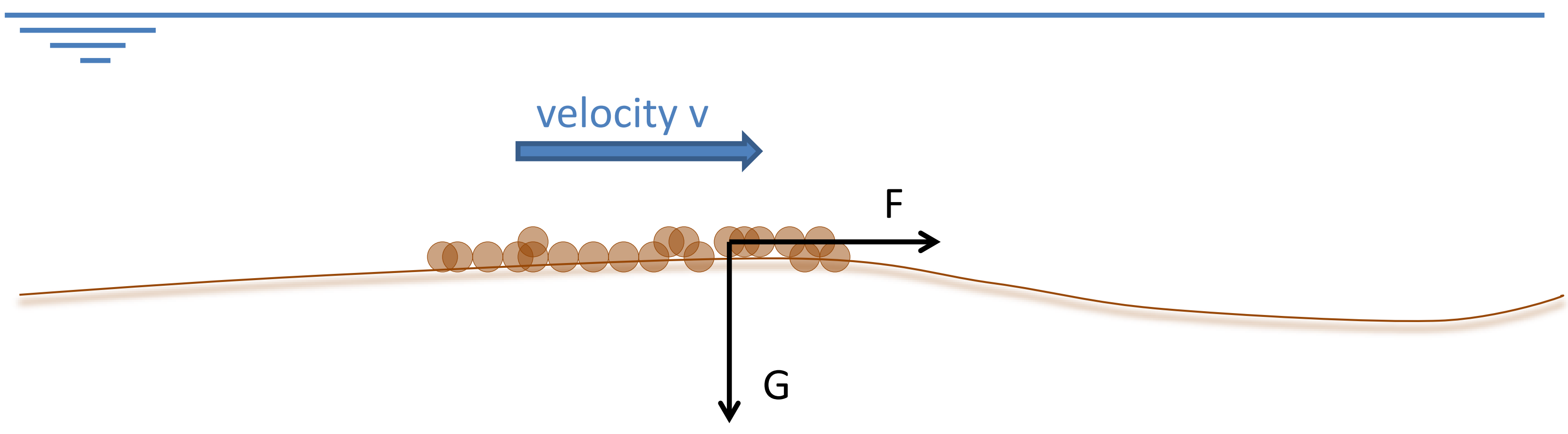
Erosion time scales

- DUNE EROSION
  - Due to storm surges
  - Fast process, up to 100 m^3/m in a few hours
- STRUCTURAL EROSION
  - Due to processes on engineering timescales
  - Up to 50 m^3/m per year
- LONG TERM EROSION
  - Processes on geological timescales (e.g. sea level rise (IPCC: up to 2m SLR by 2100), land subsidence)



Surge level
MSL
after
before
FLORIDA
Surge level
MSL
after
before
HOLLAND

General aspects transport



Velocity → bottom shear stress τ → tractive force on particle F → motion of particle

$$\left.\begin{aligned} F &= A\tau_c D^2 \\ G &= B(\rho_s - \rho) g D^3 \end{aligned}\right\} F/G$$

D = particle diameter [m]
ρ_s= density sediment [kg/m^3]
ρ = density water [kg/m^3]
g = gravity accelaration [m/s^2]

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General aspects transport

Shields parameter

$$F/G = C\frac{\tau_c}{\Delta\rho g D}, \text{ where } \Delta = \frac{\rho_s - \rho}{\rho}$$

Movement if :

F/G > tan ϕ

for spheres: $\frac{\tau_c}{\Delta\rho g D}$ > 2/3 tan ϕ

ϕ = internal angle of friction

Critical shear stress (Soulsby, 1997)

$$\theta_{cr} = \frac{\tau_{cr}}{\rho g \Delta D_{50}}$$

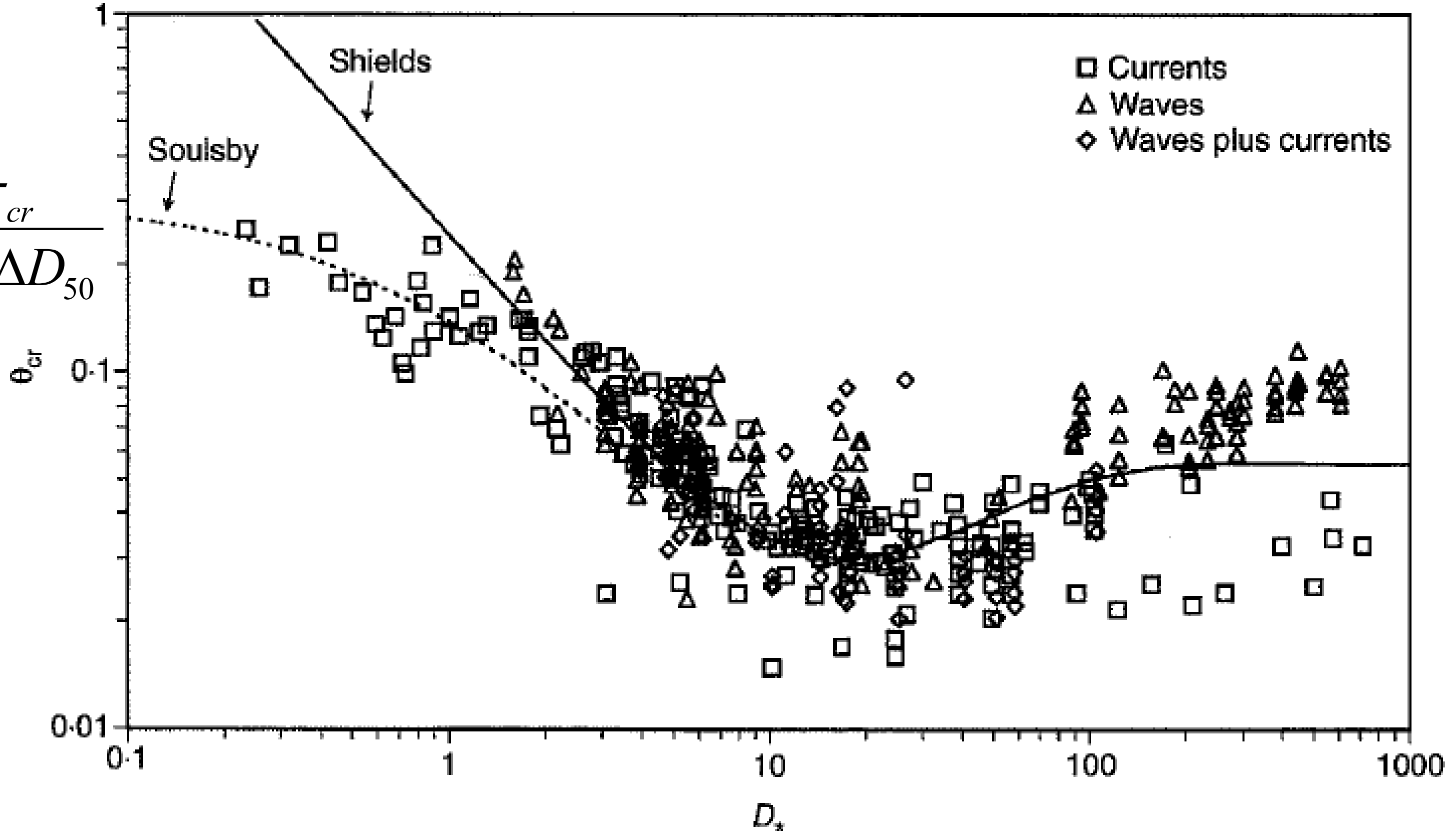

Figure 20. Threshold of motion of sediments beneath waves and/or currents

$$D_* = \left[\frac{g\Delta}{\nu^2}\right]^{1/3} D_{50}$$

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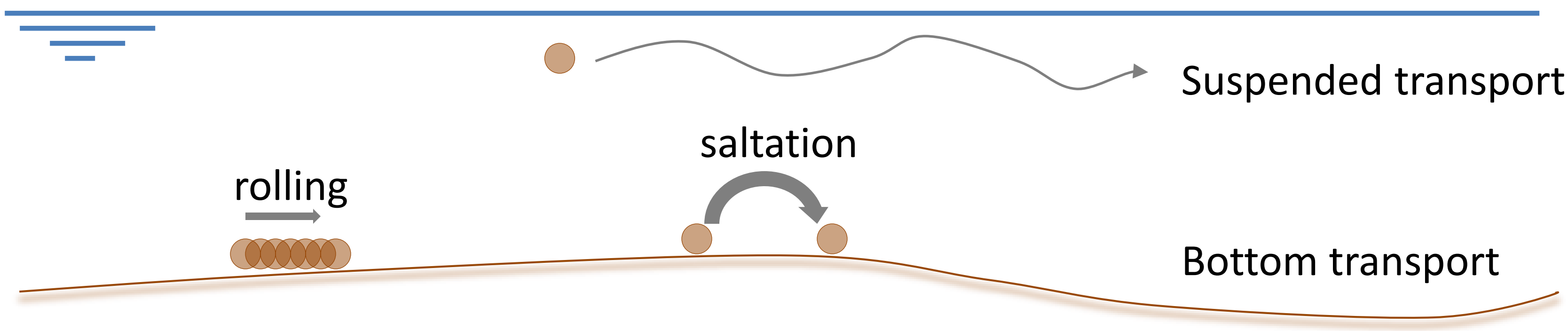
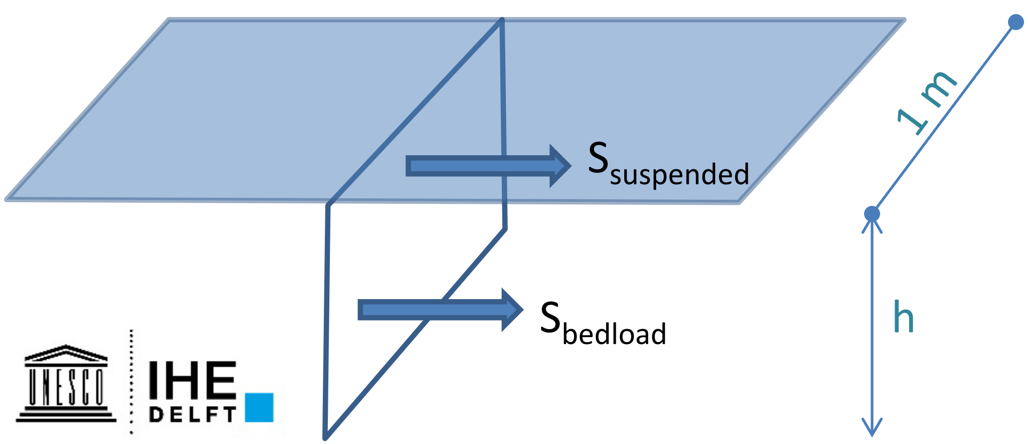
General aspects of sediment transport

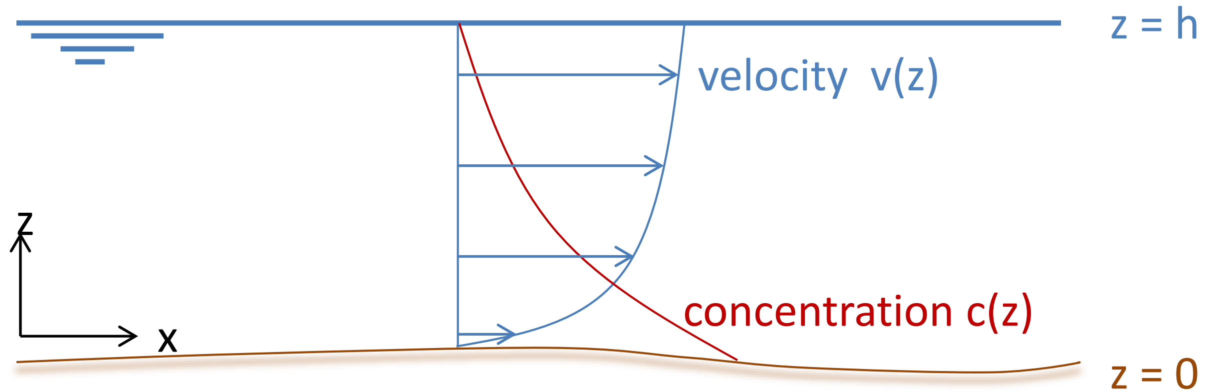


$$S_{total} = S_{suspended} + S_{bottom}$$



Transport through the plane?

Suspended sediment transport



$$S_{suspended} = \int_0^h v(z)c(z)dz$$

$$v(z) = \frac{v_*}{\kappa} \ln \frac{z}{z_0}$$

v_* = shear velocity (=sqrt(τ/ρ))

κ= von Karman's constant (=0.4)

z_0 = bed roughness length scale

logarithmic velocity profile

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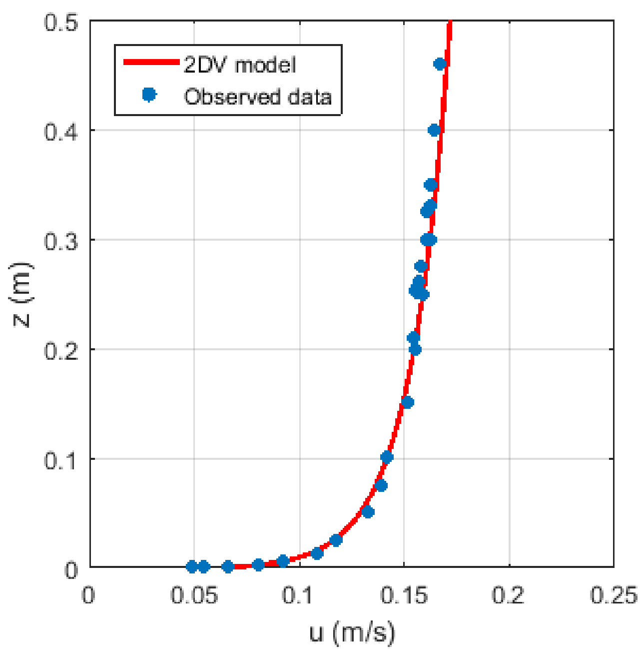
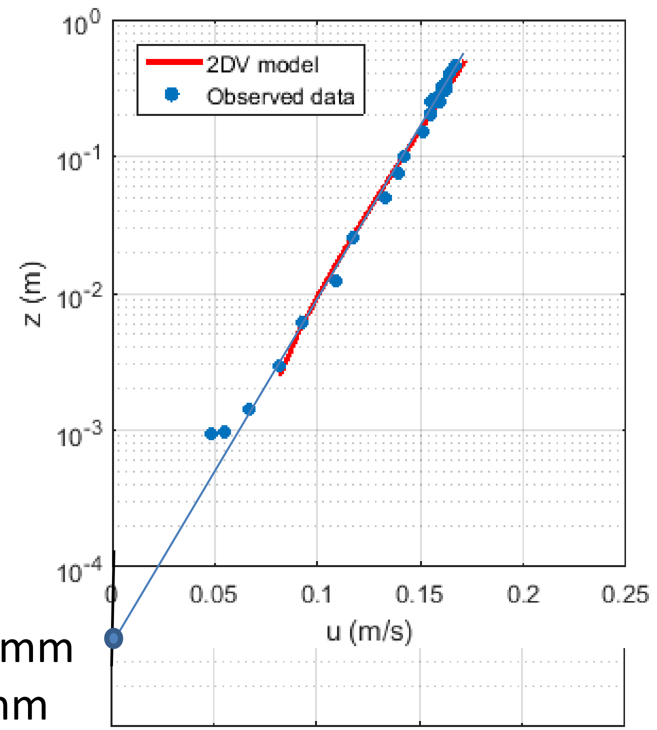
Velocity profile, current only



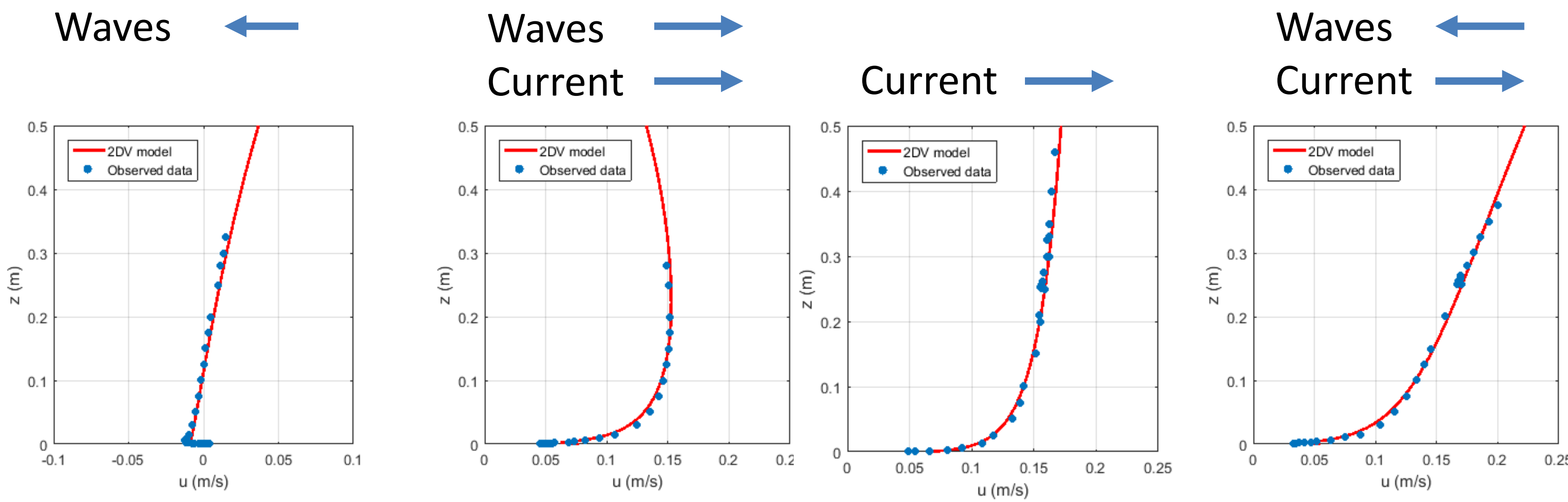
Linear scale



z_0=0.04 mm
k_s=1.2 mm

Logarithmic scale

Velocity profile, waves vs current



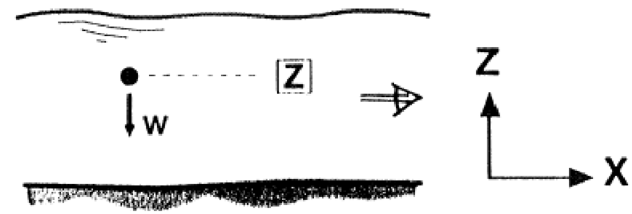
Data: Klopman (1993)

Model: 2DV wave-current interaction, Duoc Nguyen et al, 2020

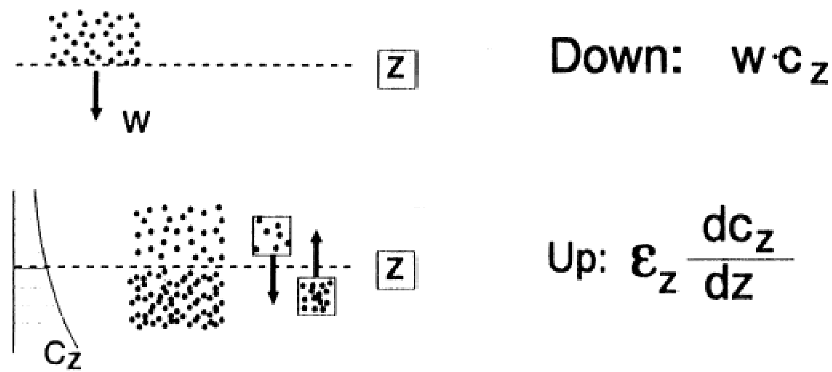
- Wave-only current is weak
- Waves have big influence on current profile for case of current plus waves

Concentration profile

- Steady state, uniform
- Turbulence: exchange of fluid and sediment



- c_z: concentration as function of z
- ε_z : eddy viscosity
  diffusion coefficient
  mixing coefficient



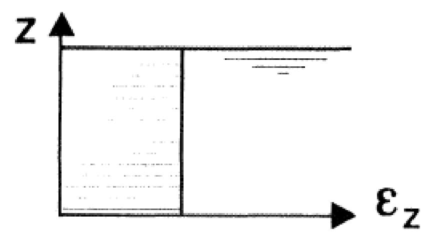
$$w \cdot c_z \downarrow + \; \varepsilon_z \frac{\partial c_z}{\partial z} \uparrow = 0$$

$$c_z = c_a \exp\left[-w \int_a^z \frac{\partial z}{\varepsilon_z} \right]$$

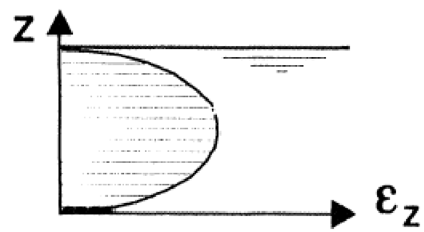
- c_z related to c_a (integration const.)
- c_z also related to 'choice' of ε_z

Possible 'choices':

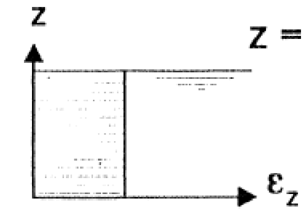
1) ε_z = constant (Coleman)



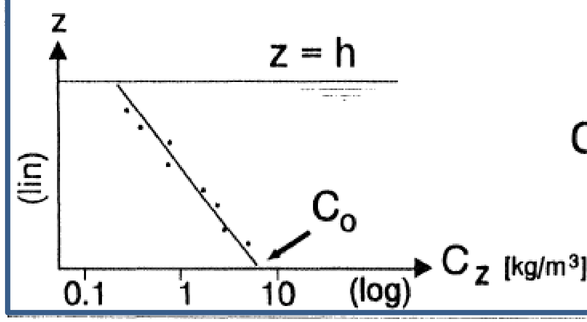
2) ε_z = parabolic (Rouse / Einstein)



Coleman:

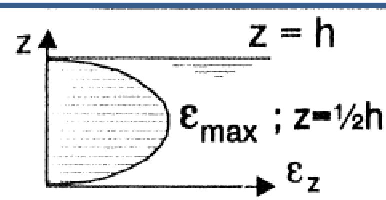


$$\varepsilon_z = \text{constant} = \varepsilon$$

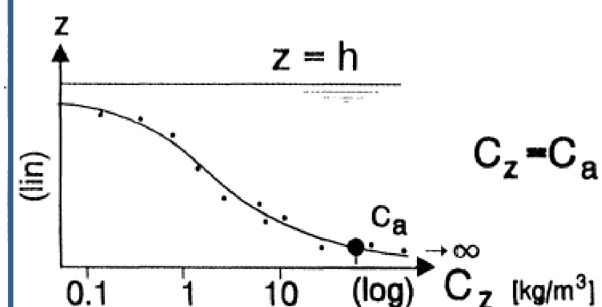


$$C_z = C_o \exp\left(\frac{-w \cdot z}{\varepsilon}\right)$$

Rouse / Einstein:



$$\varepsilon_z = 4\,\varepsilon_{max}\,\frac{z}{h}\,\frac{(h-z)}{h}$$



$$C_z = C_a\left[\frac{h-z}{z}\cdot\frac{a}{h-a}\right]^{\frac{wh}{4\varepsilon_{max}}}$$

$c\,(\,a\,)$:

- close to the bed
- a : top of bottom transport layer
- bottom transport formula and calculate $c\,(\,a\,)$

$$c\,(\,a\,) = \frac{S_b}{a \;\; \bar{v} \text{ in layer}}$$

a in coastal engineering: r

(bottom roughness)

General expression bed load transport

$$S_b \sim \sqrt{\Delta g D_{50}^3}\,\theta^{b/2}\left(m\theta - n\theta_{cr}\right)^{c/2}\left(1-\alpha\frac{\partial z_b}{\partial s}\right)$$

- Meijer-Peter and Muller (*b=0, c=3, m=1,n=1)*
- Van Rijn (1984) *(b=0,c=3-4,m=1,n=1)*

Coastal Sediment Transport
Longshore Transport and Coastline Modelling

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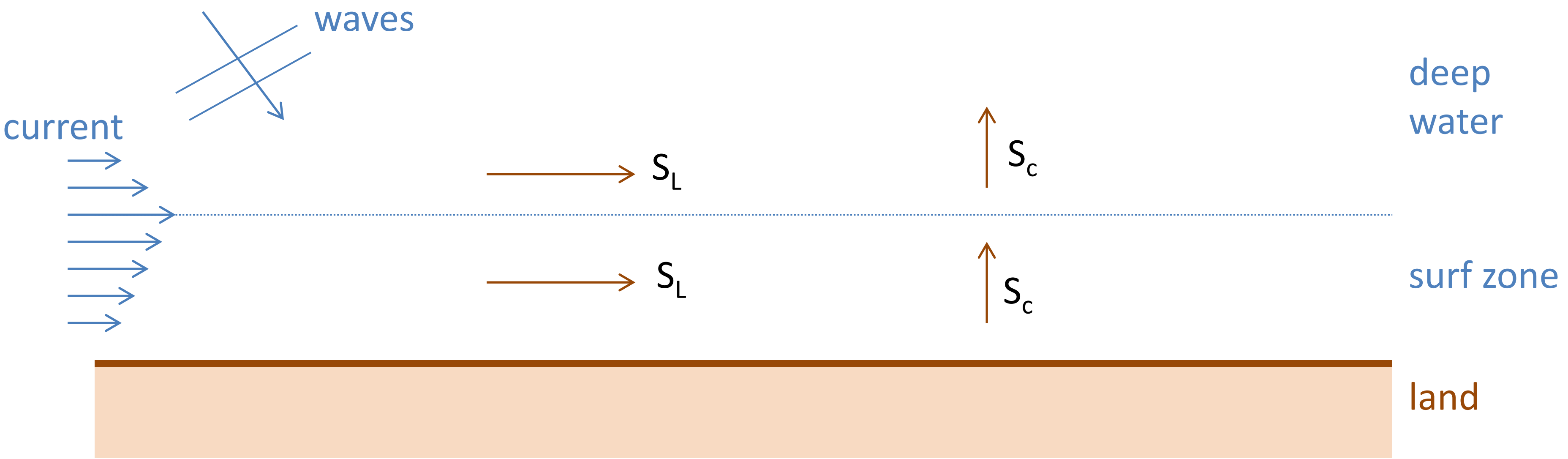


$$S_L = \frac{1}{t_0} \int_0^{t_0} \int_0^h v(z,t)\, c(z,t)\, dz\, dt$$

$$S_L = \int_0^h \left(\bar{v}\bar{c} + \cancel{\tilde{v}\tilde{c}} \right) dz$$

$$S_L \approx \int_0^h \overline{v(z)}\; \overline{c(z)}\, dz$$

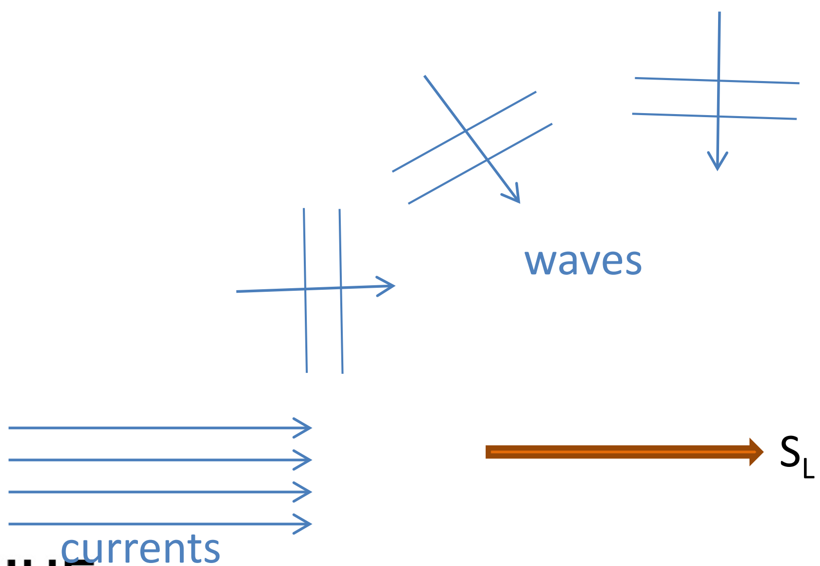
Longshore: time-averaged concentration over depth

* time-averaged velocity over depth

Longshore sediment transport

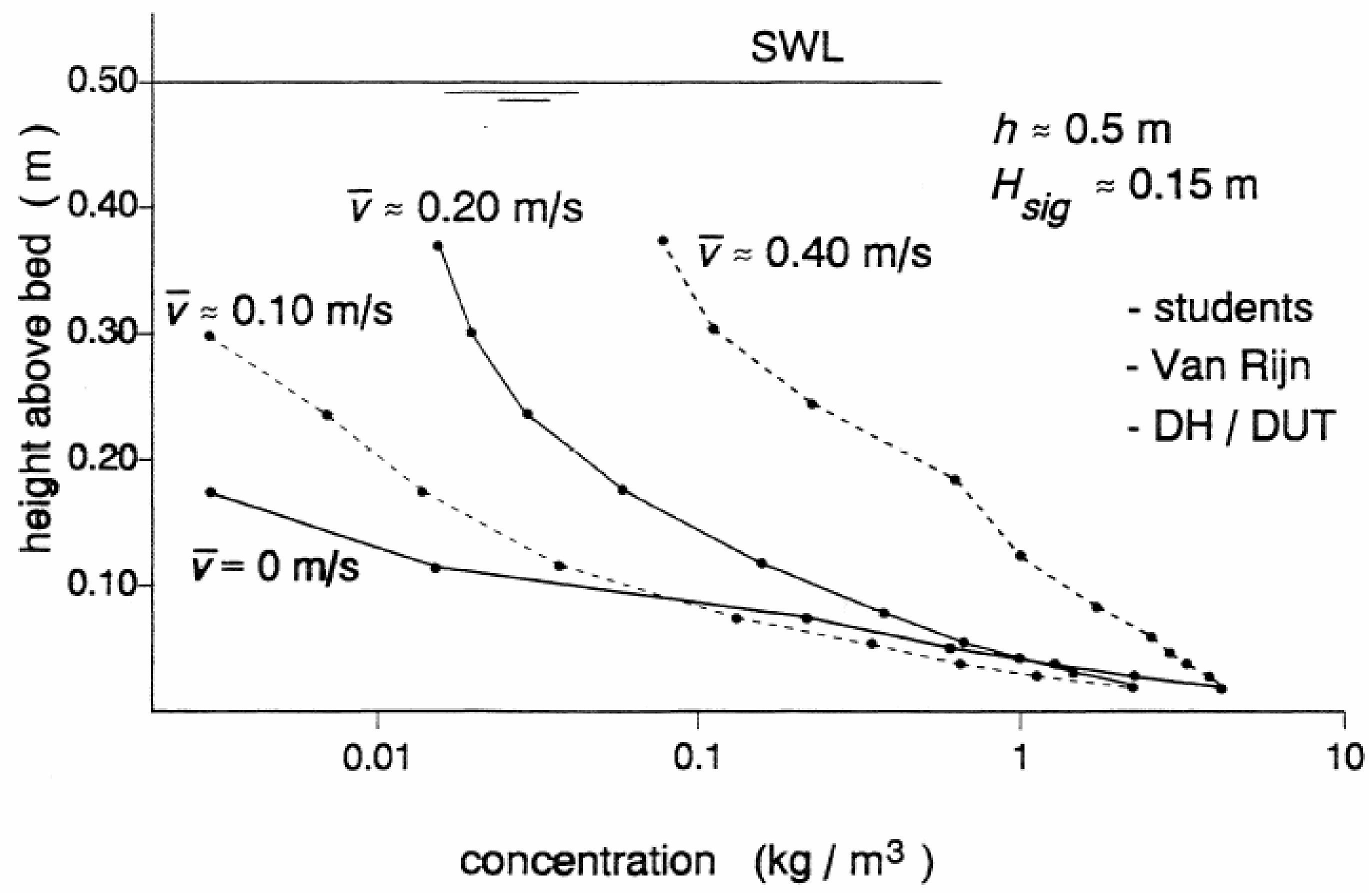
- Needed:
  - v(z) distribution;
  - $\overline{c(z)}$ distribution:

    reference concentration + distribution over depth;



Effect of angle wave and currents

Concentration profiles due to waves and current



Longshore transport

- Example: Bijker-formula (1967, 1971)
  van Rijn (1984, 1993, …,2007)

  - bottom transport $\rightarrow$ reference concentration;
  - $\overline{c(z)}$ and $\overline{v(z)}$ distributions are taken into account;
  - $\overline{c(z)}$ and $\overline{v(z)}$ same 'mechanisms' (parabolic ε_f and ε_s distribution)
  - near-bed concentration dominated by wave boundary layer

Soulsby – van Rijn formula

- very simple expression
- easy to implement
- reasonably close to Van Rijn’s full formulations
- gives clear insight in mechanisms
- bed load + suspended load
- current plus waves
- critical velocity
- bed slope effect

The root-mean-square orbital velocity is computed as

$$U_{rms} = \frac{1}{\sqrt{2}} \frac{\pi H_{rms}}{T_p \sinh(kH)}$$

water depth

Furthermore, D* is defined as (Soulsby, p.104):

$$D_* = \left[\frac{g\Delta}{\nu^2} \right]^{1/3} D_{50}$$

kinematic viscosity

relative density

$$U_{cr} = \begin{cases} 0.19 D_{50}^{0.1} \log_{10}(4H / D_{90}) & \text{if } D_{50} \leq 0.5 \text{ mm} \\ 8.5 D_{50}^{0.6} \log_{10}(4H / D_{90}) & \text{if } 0.5 < D_{50} \leq 2 \text{ mm} \end{cases}$$

$$S_{bx} = A_{cal} A_{sb} u \xi$$

$$S_{by} = A_{cal} A_{sb} v \xi$$

$$S_{sx} = A_{cal} A_{ss} u \xi$$

$$S_{sy} = A_{cal} A_{ss} v \xi$$

u, v depth-averaged velocity

where

A_{cal} a user-defined calibration factor

A_{sb} bed-load multiplication factor

$$A_{sb} = 0.05 H \left(\frac{D_{50} / H}{\Delta g D_{50}} \right)^{1.2}$$

A_{ss} suspended load multiplication factor

$$A_{ss} = 0.012 D_{50} \frac{D_*^{-0.6}}{\left(\Delta g D_{50} \right)^{1.2}}$$

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ξ a general multiplication factor

$$\xi = \left(\sqrt{U^2 + \frac{0.018}{C_D} U_{rms}^2} - U_{cr} \right)^{2.4}$$

$$C_D = \left[\frac{\kappa}{\ln(H / z_0) - 1} \right]^2$$

where z_0 equals 6 mm and the Von Karman constant κ is set to 0.4.

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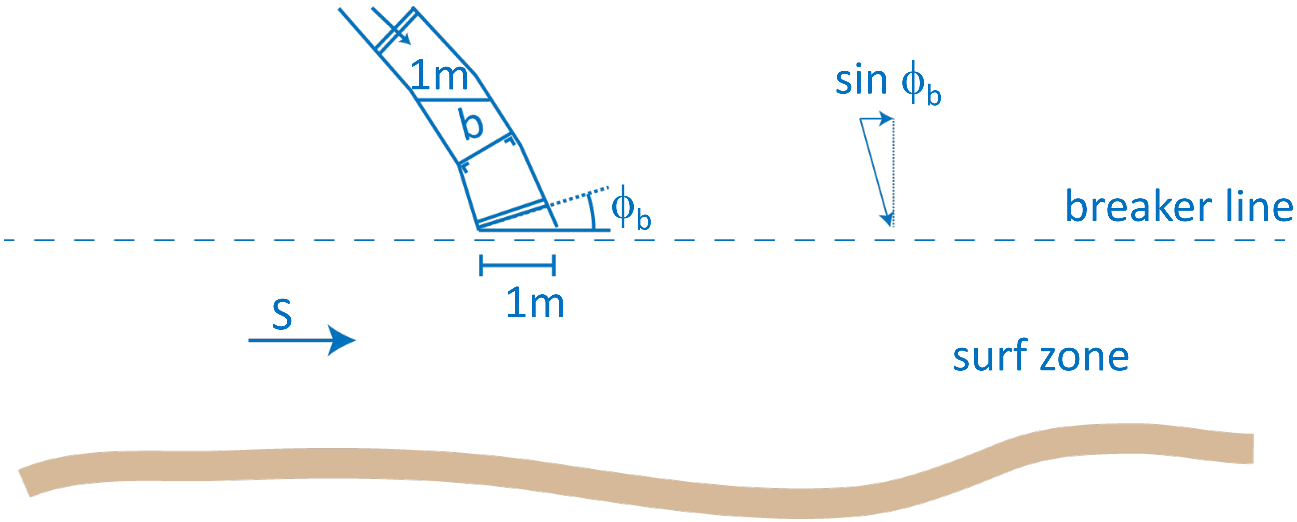
Exercise about lectures 1 and 2

- V=0.7 m/s
- H=1 m
- h=2 m
- T=7 s
- D50=0.2 mm; D90=0.3 mm
- r=0.05 m
- Compute $\tau_c, u_0, f_w, \tau_w, \tau_{cw}, S_b, S_s$

Longshore sediment transport (bulk)

- CERC formula (SPM 1984)
  - Sandy environments only;
  - Transport determined by longshore wave energy flux P_L;
  - Parameters determined at the breaker line!
  - Original formulation: H_{rms}, not H_s



Energy flux between wave orthogonals P = E n c b

Longshore flux at the breakerline $P_l = E_b\, n_b\, c_b \cos\phi_b \sin\phi_b$

$S = A\, P_l$, where A is not dimensionless!

If we substitute energy flux for wave height ($E = 1/8\, \rho g H^2$), we get:

$S = B\, H_b^2\, n_b\, c_b \sin\phi_b \cos\phi_b$, where B (≈0.04) is dimensionless.

S = volumetric sediment transport [m^3/s]

nc = wave group celerity [m/s]

ϕ= wave angle [-]

H = wave height [m]

CERC formula

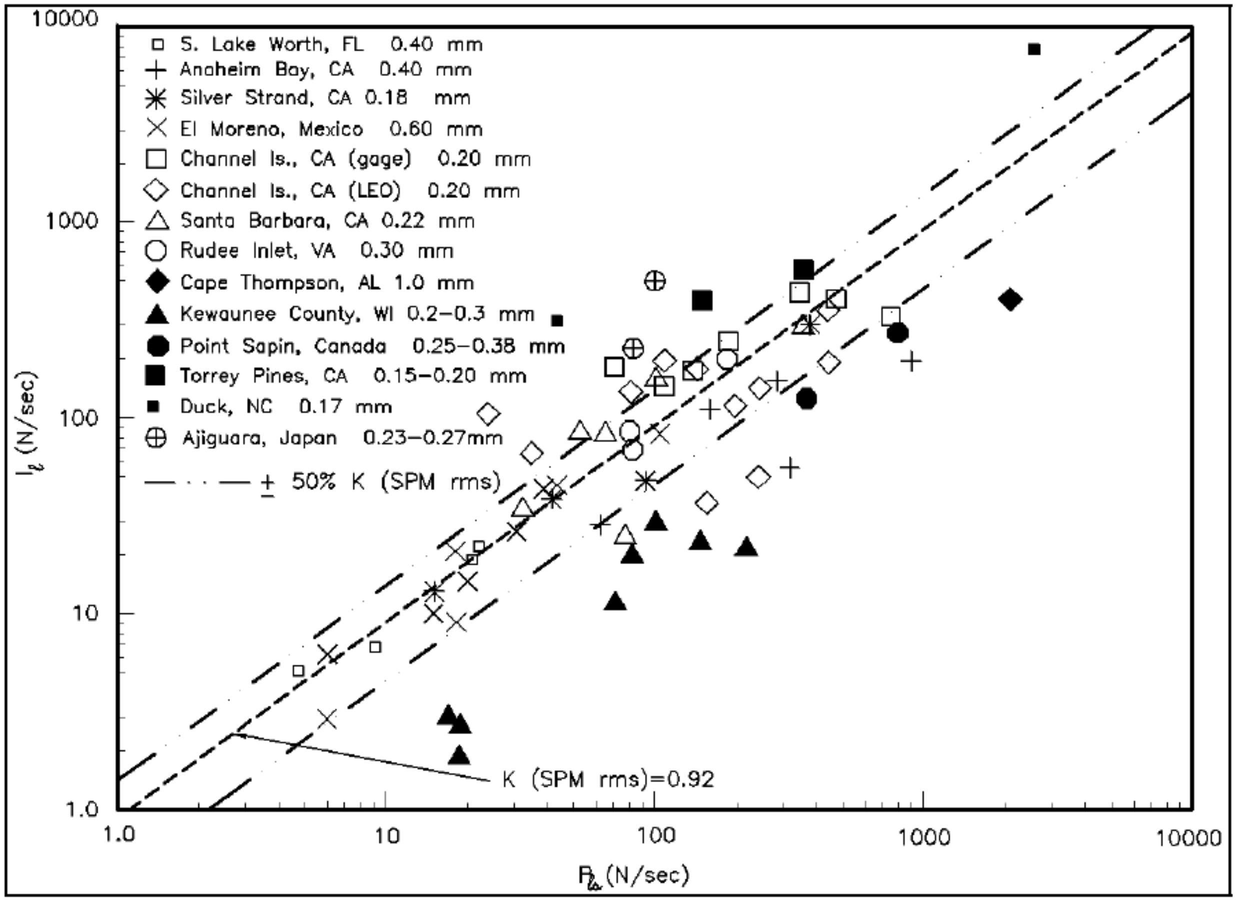


Figure III-2-4. Field data relating I_ℓ and P_ℓ

Example

H_{0S} = 1m; T = 7s; ϕ_0=20°

$$\gamma = \frac{H_b}{h_b} = 0.7$$

Determine breaker height:

- first estimate $H_b=H_0$
- calculate breaker depth based on $h_b=H_b/\gamma$,
- calculate H_b using Snell's law, Ks and Kr
- repeat last two steps until h_b does not change anymore

Here: $\varphi_0 = 20^o$, $\varphi_b = 7.2^o$; S = 0.03 m^3 / s = 810,000 m^3 / yr

Worked out in excel

rho	1025.00	1025.00	1025.00	1025.00
g	9.81	9.81	9.81	9.81
gamma	0.70	0.70	0.70	0.70
H0	1.00	1.00	1.00	1.00
T	7.00	7.00	7.00	7.00
theta0	20.00	20.00	20.00	20.00
B	0.04	0.04	0.04	0.04
E0	1256.91	1256.91	1256.91	1256.91
C0	10.93	10.93	10.93	10.93
Cg0	5.46	5.46	5.46	5.46
sin(theta0)	0.34	0.34	0.34	0.34
cos(theta0)	0.94	0.94	0.94	0.94
hb	1.43	1.68	1.61	1.63
C	3.74	4.06	3.98	4.00
Cg	3.74	4.06	3.98	4.00
sin(theta)	0.12	0.13	0.12	0.13
theta	6.73	7.30	7.15	7.19
cos(theta)	0.99	0.99	0.99	0.99
Ks	1.21	1.16	1.17	1.17
Kr	0.97	0.97	0.97	0.97
Hb	1.18	1.13	1.14	1.14
hb	1.68	1.61	1.63	1.63
S m3/s	0.02	0.03	0.03	0.03
S Mm3/yr	0.76	0.82	0.81	0.81

Assignment

- Hs0=3m, T=8s, $\gamma = 0.7$
- Angle of incidence:
- 75,60,50,45,40,30,20,10 deg.
- Compute conditions at breaker line
- Compute longshore sediment transport using coefficient B=0.04

$$S = B \, H_b^2 \, n_b \, c_b \, \cos\varphi_b \, \sin\varphi_b$$

Variations of CERC formula

H_{sig}

$$S = 0.040\, H_b^2\, c_b\, n_b \sin\varphi_b \cos\varphi_b$$

$$n_b = 1$$

$$S = 0.040\, H_b^2\, c_b \sin\varphi_b \cos\varphi_b$$

$$H^2 n\, c\, b = \text{const}$$

$$S = 0.040\, H_o^2\, c_o\, n_o \sin\varphi_b \cos\varphi_o$$

$$n_o = ½$$

$$S = 0.020\, H_o^2\, c_o \sin\varphi_b \cos\varphi_o$$

Snel's law: $\frac{c_o}{c_b} = \frac{\sin\varphi_o}{\sin\varphi_b}$

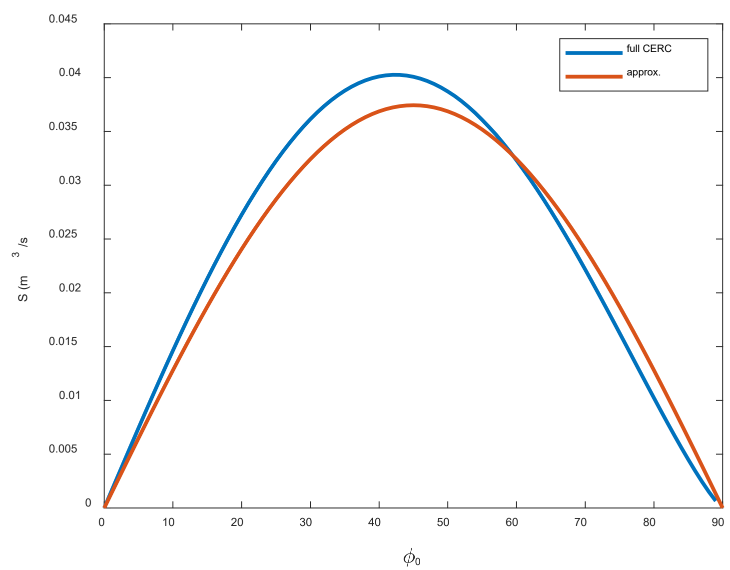
$$S = 0.020\, H_o^2\, c_b \sin\varphi_o \cos\varphi_o$$

$$\sin\varphi_o \cos\varphi_o = ½ \sin(2\varphi_o)$$

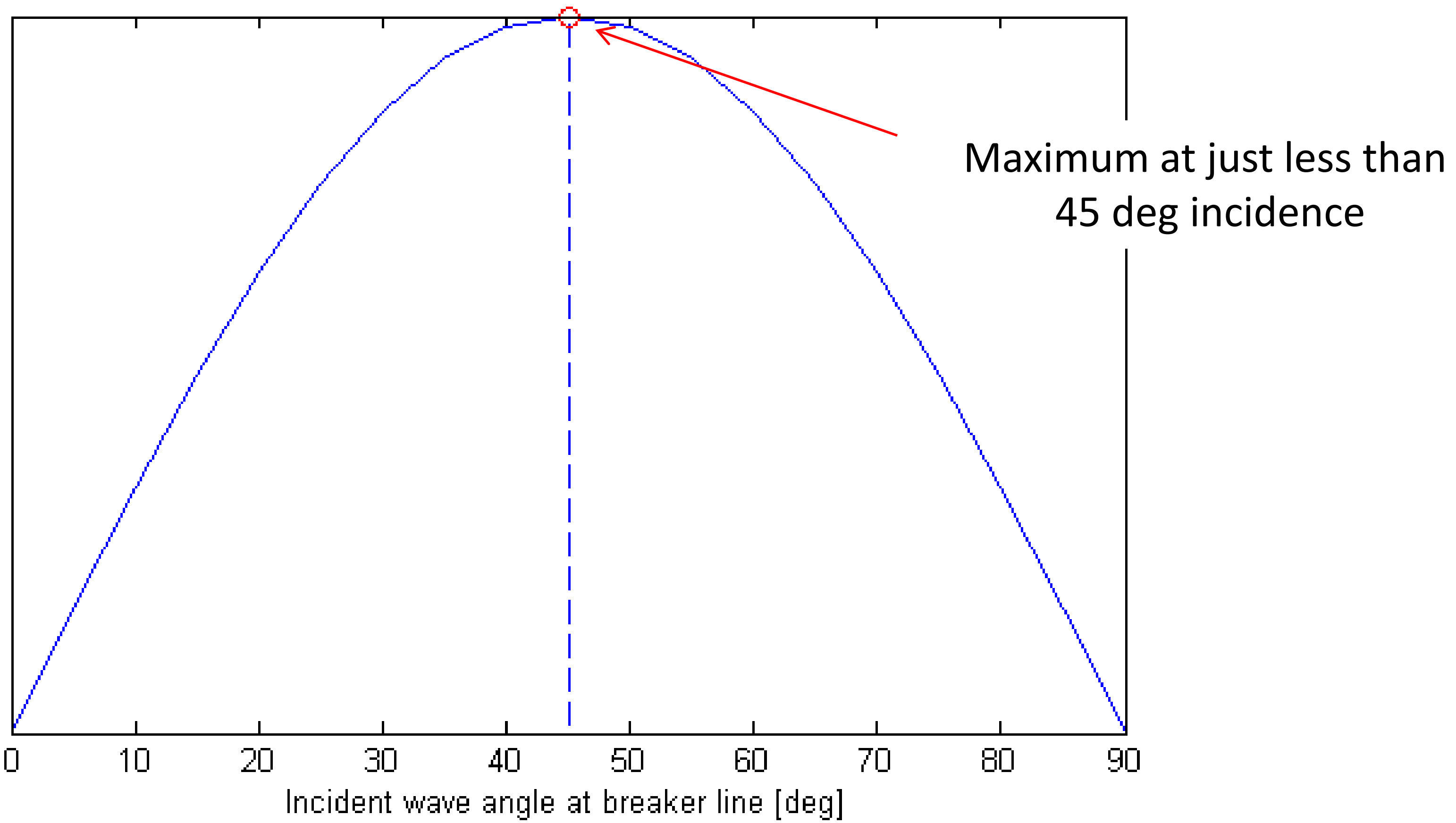
$$S = 0.010\, H_o^2\, c_b \sin(2\varphi_o)$$

$$c_b = \sqrt{g h_b} \approx \sqrt{g/\gamma}\sqrt{H_b}$$

$$S = 0.01\sqrt{g/\gamma}\, H_0^{2.5} \sin(2\varphi_0)$$



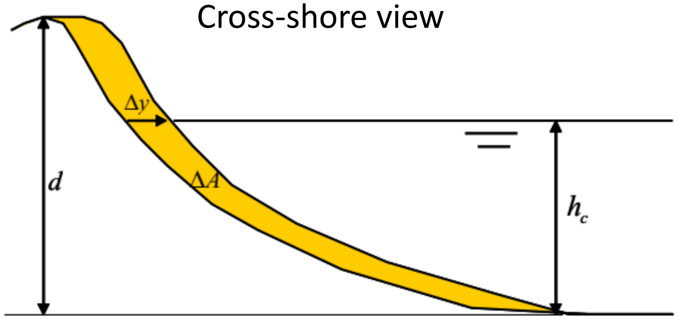
Longshore sediment transport



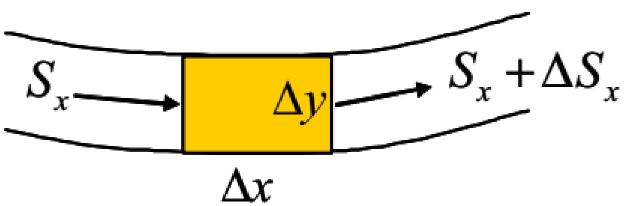
If incident waves deviate little from shore normal => S≈0

Coastline modeling

Cross-shore view



Longshore view



Volume change in time:

$$\begin{aligned} \Delta V &= \Delta A \Delta x \\ &= d \Delta y \Delta x = -\Delta S_x \Delta t \end{aligned}$$

Coastline modeling

- Change in volume over time ΔV:

$$\Delta V = \Delta A \Delta x = d \Delta y \Delta x = -\Delta S_x \Delta t \Rightarrow \quad \frac{\Delta y}{\Delta t} = -\frac{1}{d}\frac{\Delta S_x}{\Delta x}$$

$$\lim_{\Delta x \to \infty} \Rightarrow \frac{\partial y}{\partial t} = -\frac{1}{d}\frac{\partial S_x}{\partial x} \qquad (1)$$

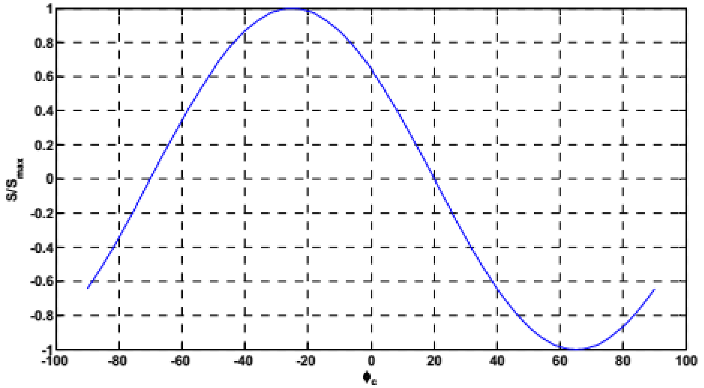
- We now need an expression for the transport gradient in function of the coastline tangent Δy/Δx

Coastline modeling

- For small angles of wave incidence relative to the coastline orientation:

$$S_x \approx -s_x \varphi_c + S_{x,0} = -s_x \arctan \frac{\partial y}{\partial x} + S_{x,0} \approx -s_x \frac{\partial y}{\partial x} + S_{x,0}$$

where s_x is the so-called *coastal constant*, and $S_{x,0}$ is the transport under a coastline angle of 0 degrees.



Differentation wrt x yields for the transport gradient:

$$\frac{\partial S_x}{\partial x} = -s_x \frac{\partial^2 y}{\partial x^2} \qquad (2)$$

Coastline modeling

- Combining (1) and (2) yields the *Pelnard-Considère diffusion equation*:

$$\frac{\partial y}{\partial t} = \frac{s_x}{d}\frac{\partial^2 y}{\partial x^2}$$

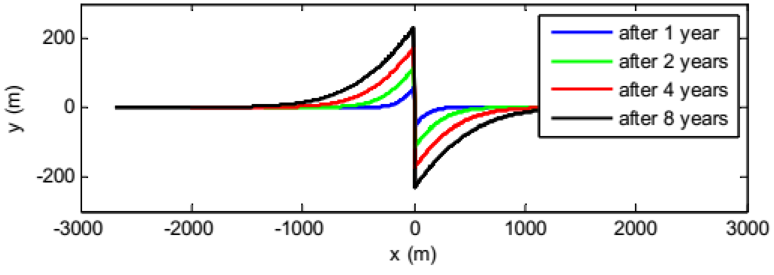
Coastline modeling

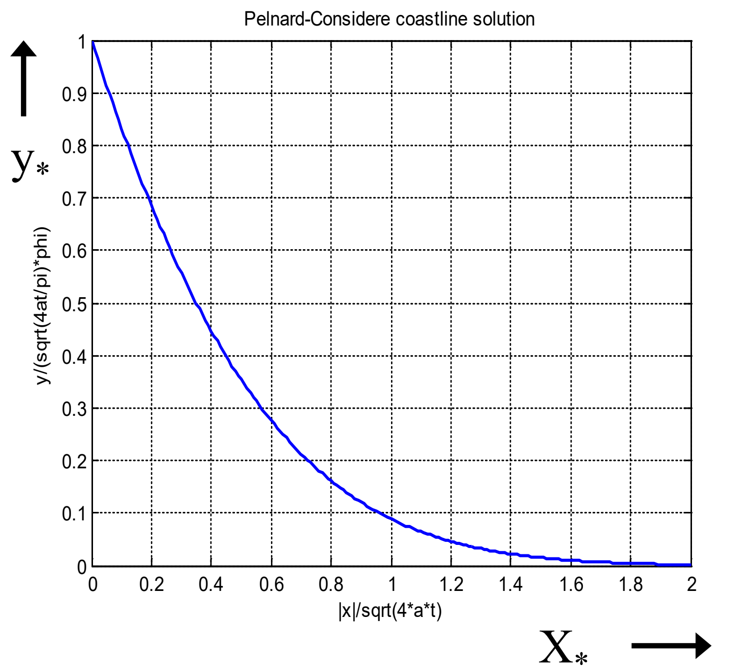
- Case of a groyne on a straight coast

$$y_* = \left[\exp\left(-x_*^2\right) - x_*\sqrt{\pi}\left(1 - erf(x_*)\right)\right] sign(x)$$

$$x_* = \frac{|x|}{\sqrt{4at}}, \quad y_* = \frac{-y}{\sqrt{4at}}\frac{\sqrt{\pi}}{\varphi'}, \quad a = \frac{s_x}{d} = \frac{S_\infty}{\varphi' d}$$

φ' is the angle of wave incidence, S_∞ the undisturbed transport away from the structure [m^3/yr]





Example application

- At the groyne:
  - Accretion goes with square root of time and is proportional with wave angle
  - Time to fill up till tip of groyne with length L

$$x = 0 \Rightarrow x_* = 0 \Rightarrow y_* = 1$$

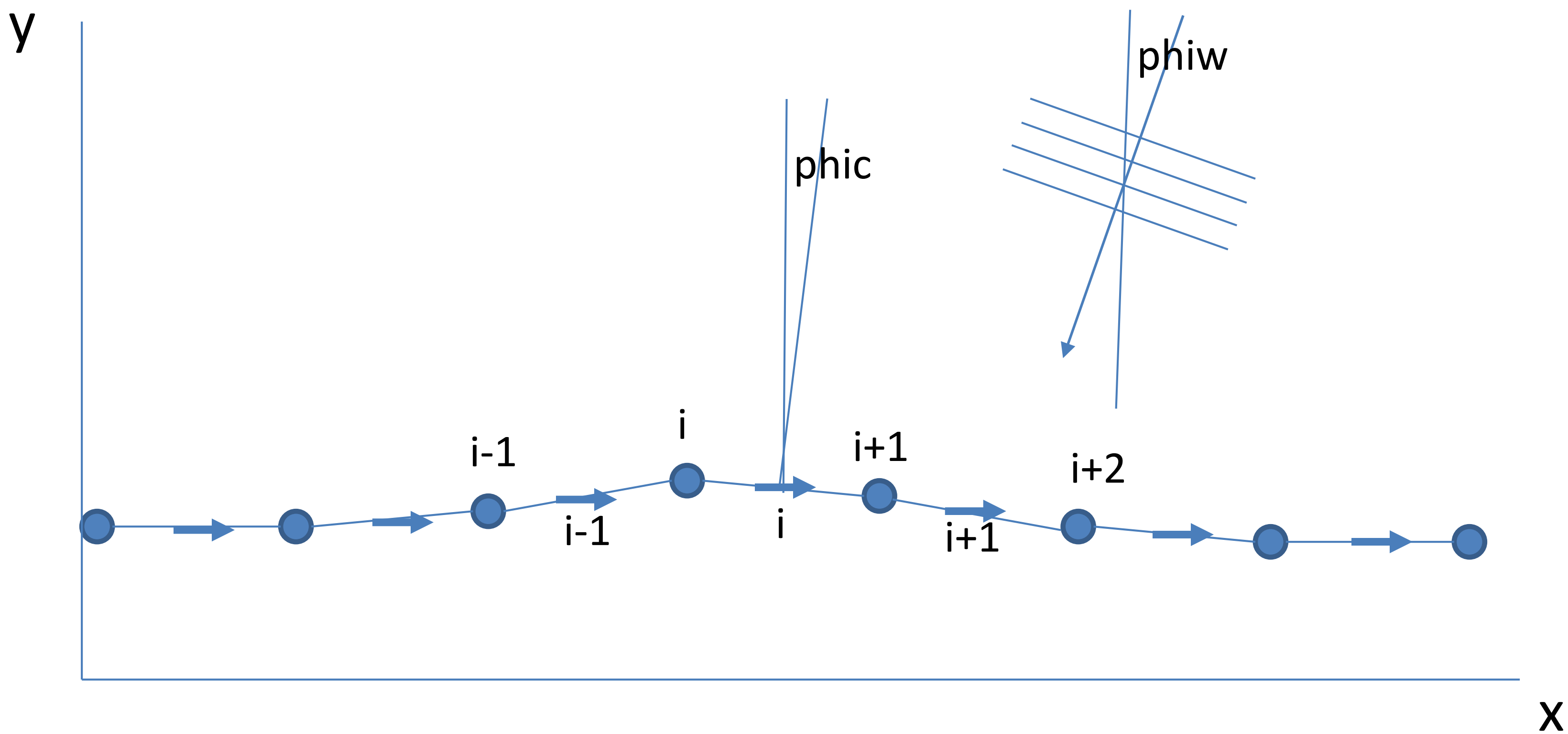
$$y = \sqrt{4at}\,\varphi' / \sqrt{\pi}$$

$$y = L \Rightarrow t = \frac{\pi}{4a\varphi^2} L^2$$

Numerical approach

- Compute S-phi curve, if necessary different S-phi curves per region or cell
- Start at given y(x)
- Compute phi(x)=arc tan (dy/dx) in each point
- Compute S(x)=f(phi(x))
- compute dS(x)/dx in each point
- compute dy/dt=-1/d*dS(x)/dx
- Compute new y = old y + dy/dt * delta t
- Staggered grid is convenient

Staggered grid



Numerical scheme

$$\varphi_{c,i} = -\tan^{-1}\frac{y_{i+1} - y_i}{\Delta x} \qquad i = 1:n-1$$

$$S_i = B\,H_s^{2.5}\sqrt{g/\gamma}\,\sin\left(2\left(\varphi_{c,i} - \varphi_w\right)\right) \qquad i = 1:n-1$$

$$dSdx_i = \frac{S_i - S_{i-1}}{\Delta x} \qquad i = 2:n-1$$

$$dydt_i = -\frac{1}{d}\,dSdx_i \qquad i = 2:n-1$$

$$y_i^{t+\Delta t} = y_i^t + dydt_i\,\Delta t \qquad i = 2:n-1$$

$$\Delta t < \frac{d\Delta x^2}{4S_{\max}}$$

Assignment 3

- Assume that S=B Hs2.5 sqrt(g/gamma)sin(2 (phic-phiw)
- Hs=1m
- Incident wave angle w.r.t. coast is -30 deg.
- B=0.01
- There is a groin at x=20,000 m, infinitely long
- Compute numerically the coastline over 0-40,000 m, at t= 1,2,5,10,20 years
- Compare solution after these times with Pelnard-Considere analytical solution
- Experiment with more groins and with a nourishment at t=0
- Build in the possibility to read an arbitrary initial coastline.
- Write a brief report on the findings and include the MATLAB code