Wind-driven currents and setup

- Important for large-scale flow patterns
- Act over wide areas
- Have small horizontal gradients
- Important effect of inertia (‘memory’)
- Can be substantially modified by density gradients that reduce turbulence
- Let’s start simple: depth-averaged, longshore

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Longshore momentum balance

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial y} + fu = \frac{\partial}{\partial x}\left(D_h \frac{\partial v}{\partial x}\right) + \frac{\tau_{w,y}}{\rho h} - \frac{\tau_{by}}{\rho h} - g\frac{\partial \eta}{\partial y}$$

$$\vec{\tau}_w = \rho_a C_d \left|\vec{W}_{10m}\right| \vec{W}_{10m}$$

wind shear stress

density of air ~1.25 kg/m3

wind drag coefficient

~0.001-0.003

wind speed at 10 m above sea

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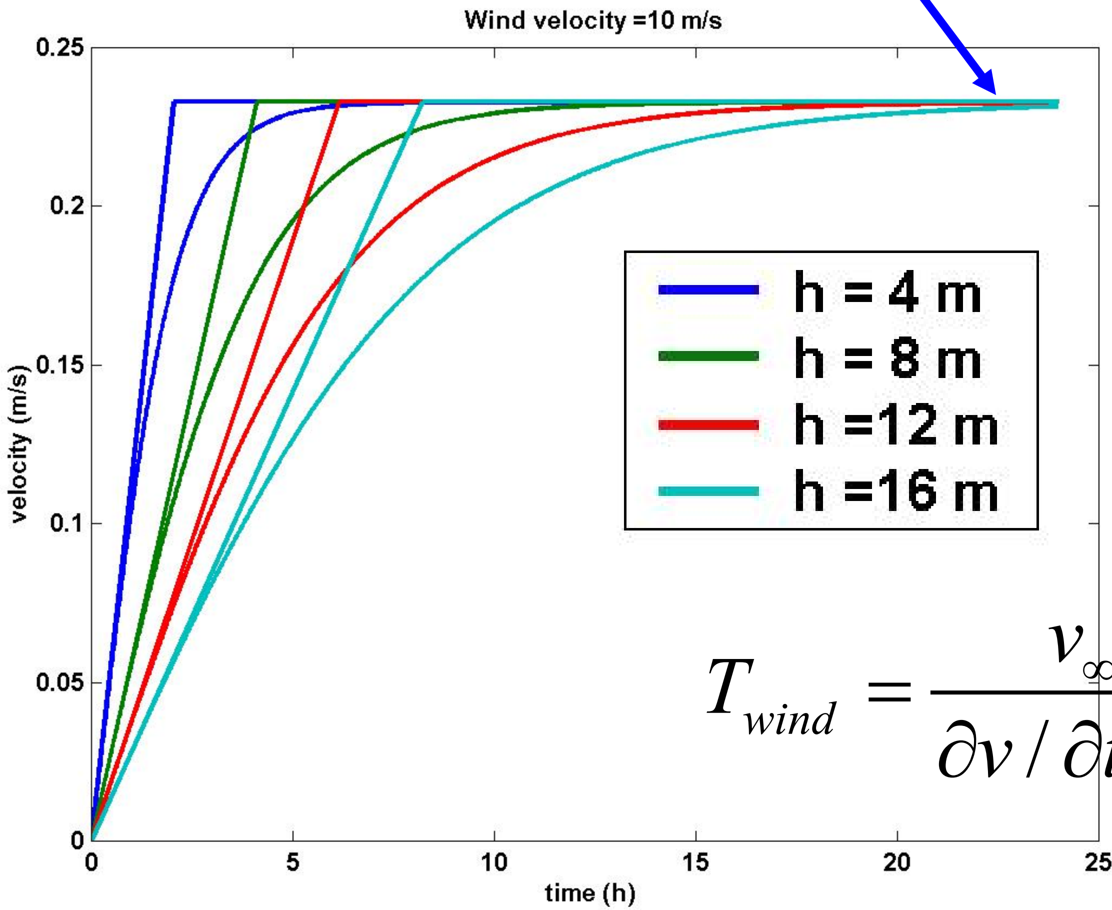
Equilibrium situation

$$\tau_{w,y} = \tau_{b,y} \Rightarrow \tau_{w,y} = \rho_w C_f |v| v$$

$$\Rightarrow v_\infty = \sqrt{\frac{\tau_{w,y}}{\rho_w C_f}} \quad , v > 0$$

$$v_\infty = \sqrt{\frac{\tau_{w,y}}{\rho_w C_f}}$$

$$\frac{\partial v}{\partial t} = \frac{\tau_{w,y}}{\rho h} - \frac{\tau_{by}}{\rho h}$$



$$T_{wind} = \frac{v_\infty}{\partial v / \partial t \,|_{t=0}} = h\sqrt{\frac{\rho_w}{\tau_w C_d}}$$

Wind-driven currents

- Longshore wind-driven current approaches uniform flow field
- Spin-up time is proportional to water depth
- It is possible to get a reasonable estimate of current speed, given:
  – time series of wind speed and direction
  – local water depth

Wind setup

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = \frac{\partial}{\partial y} D_h \frac{\partial u}{\partial y} + \frac{\tau_{w,x}}{\rho h} - \frac{\tau_{bx}}{\rho h} - g\frac{\partial \eta}{\partial x}$$

$$-fv = \frac{\tau_{w,x}}{\rho h} - g\frac{\partial \eta}{\partial x}$$

Important on open ocean

Storm surge

$$g\frac{\partial \eta}{\partial x} = \frac{\tau_{w,x}}{\rho h}$$

$$h = const \Rightarrow \frac{\partial \eta}{\partial x} = \frac{\tau_{w,x}}{\rho g h} \Rightarrow \eta = \eta_0 + \frac{\tau_{w,x}}{\rho g h}(x - x_0)$$

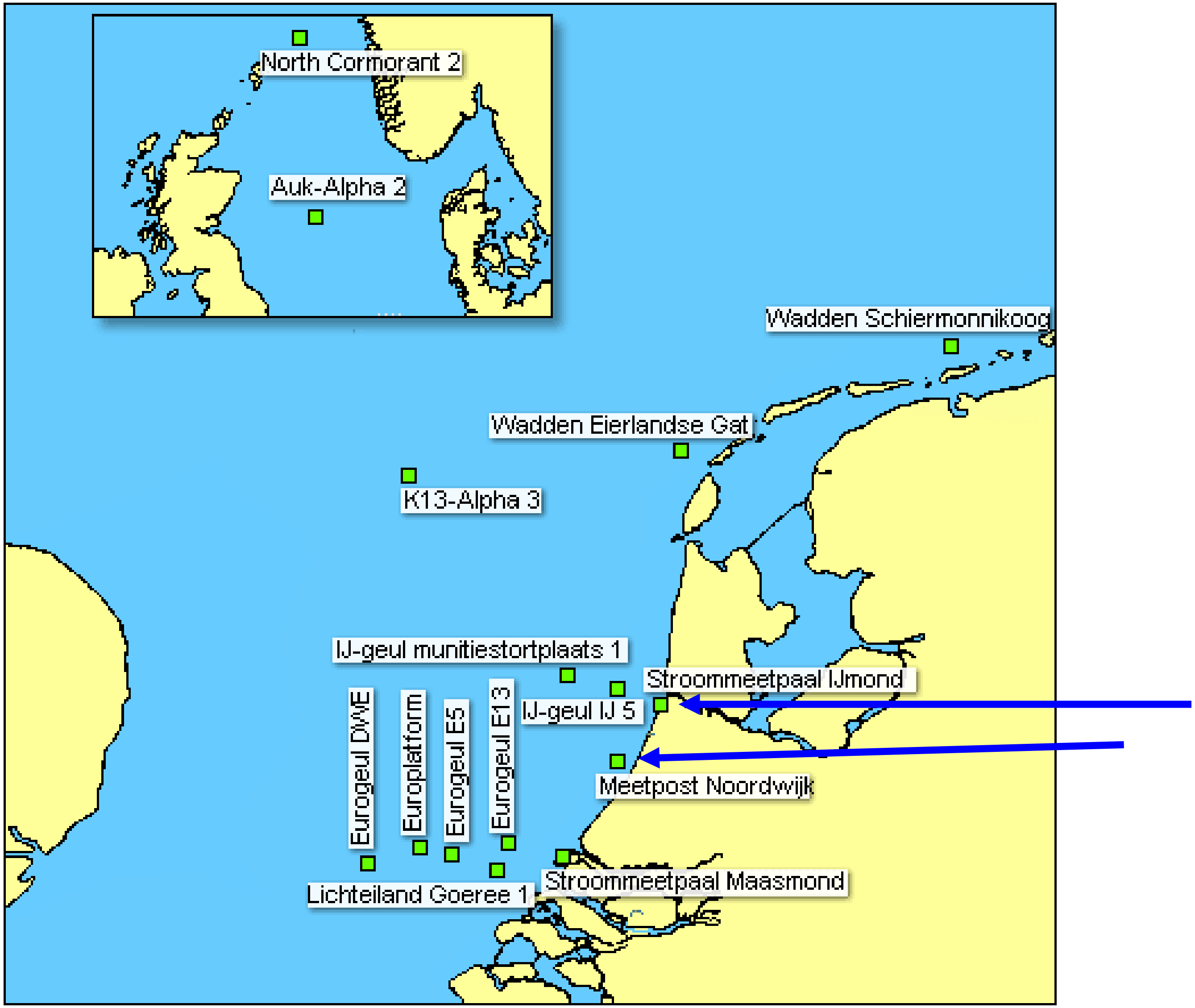
Example

- (x-x0)=250 km
- *h=25 m*
- *W=20 m/s*
- *Cd=0.002*
- ρ_a *=1.25 kg/m3*

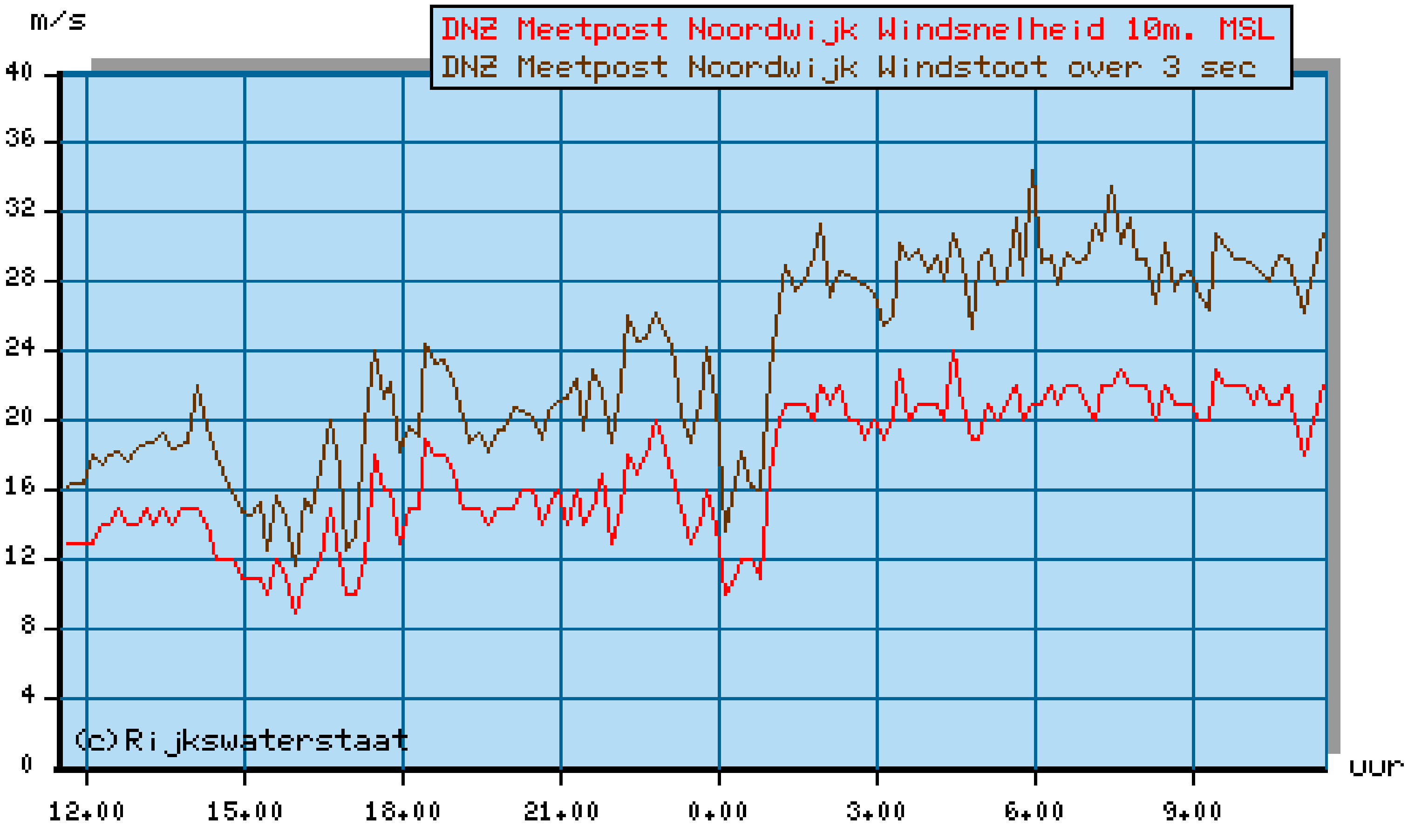
$$\vec{\tau}_w = \rho_a C_d \left|\vec{W}_{10m}\right| \vec{W}_{10m}$$

$$\eta - \eta_0 = \frac{\tau_{w,x}}{\rho g h}(x - x_0)$$

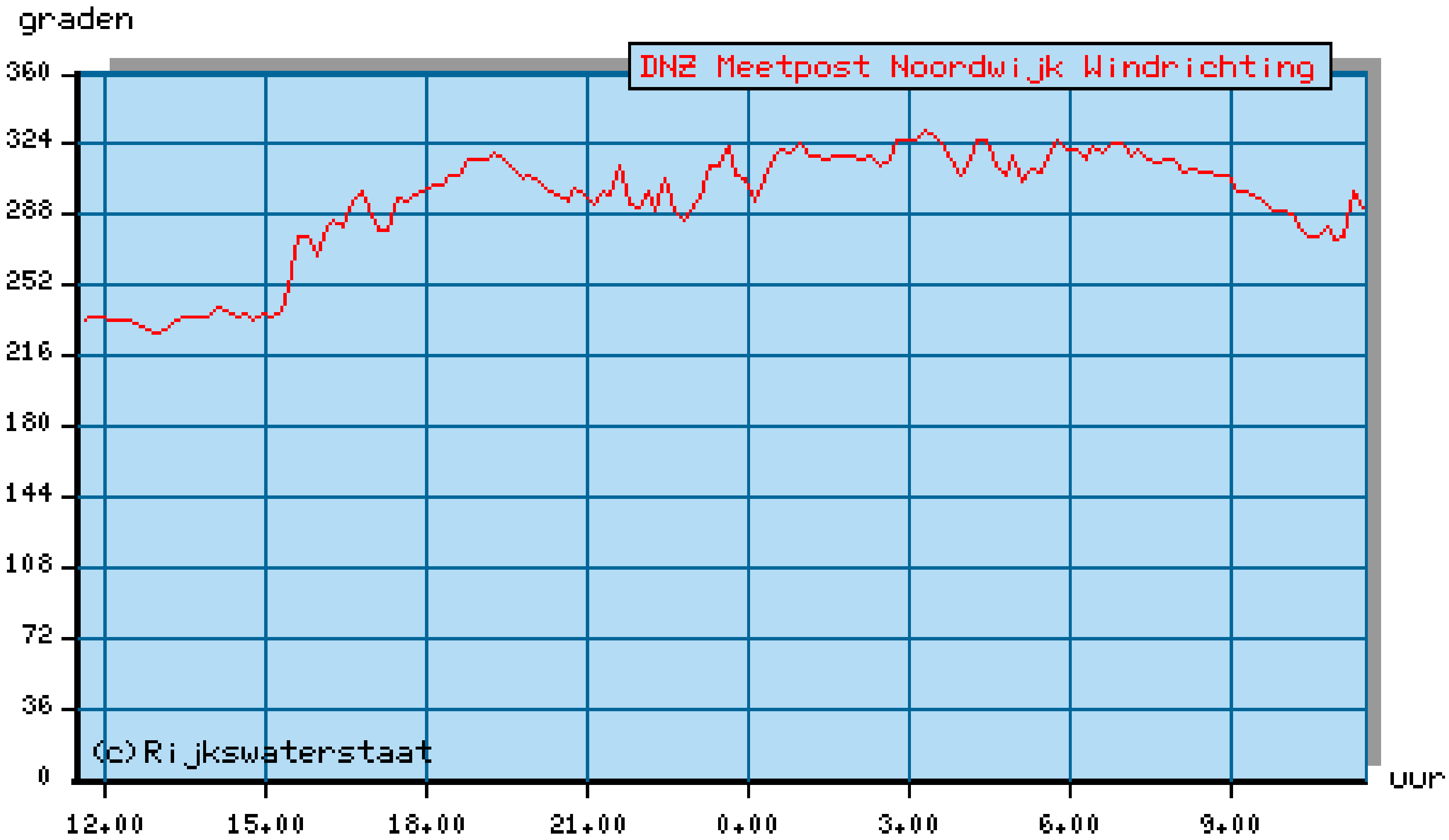
www.actuelewaterdata.nl



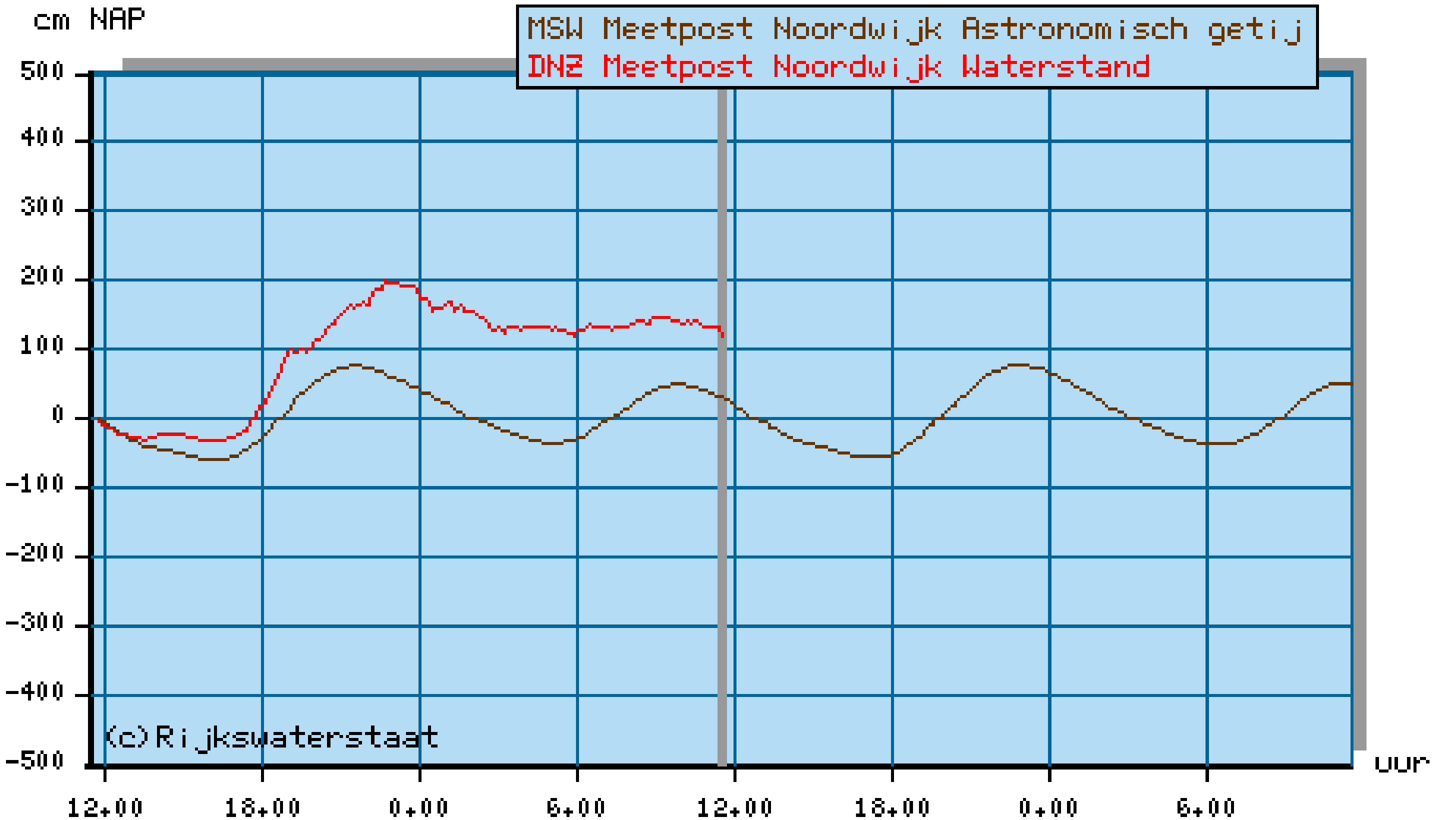
Windspeed 25/11/2005



Wind direction

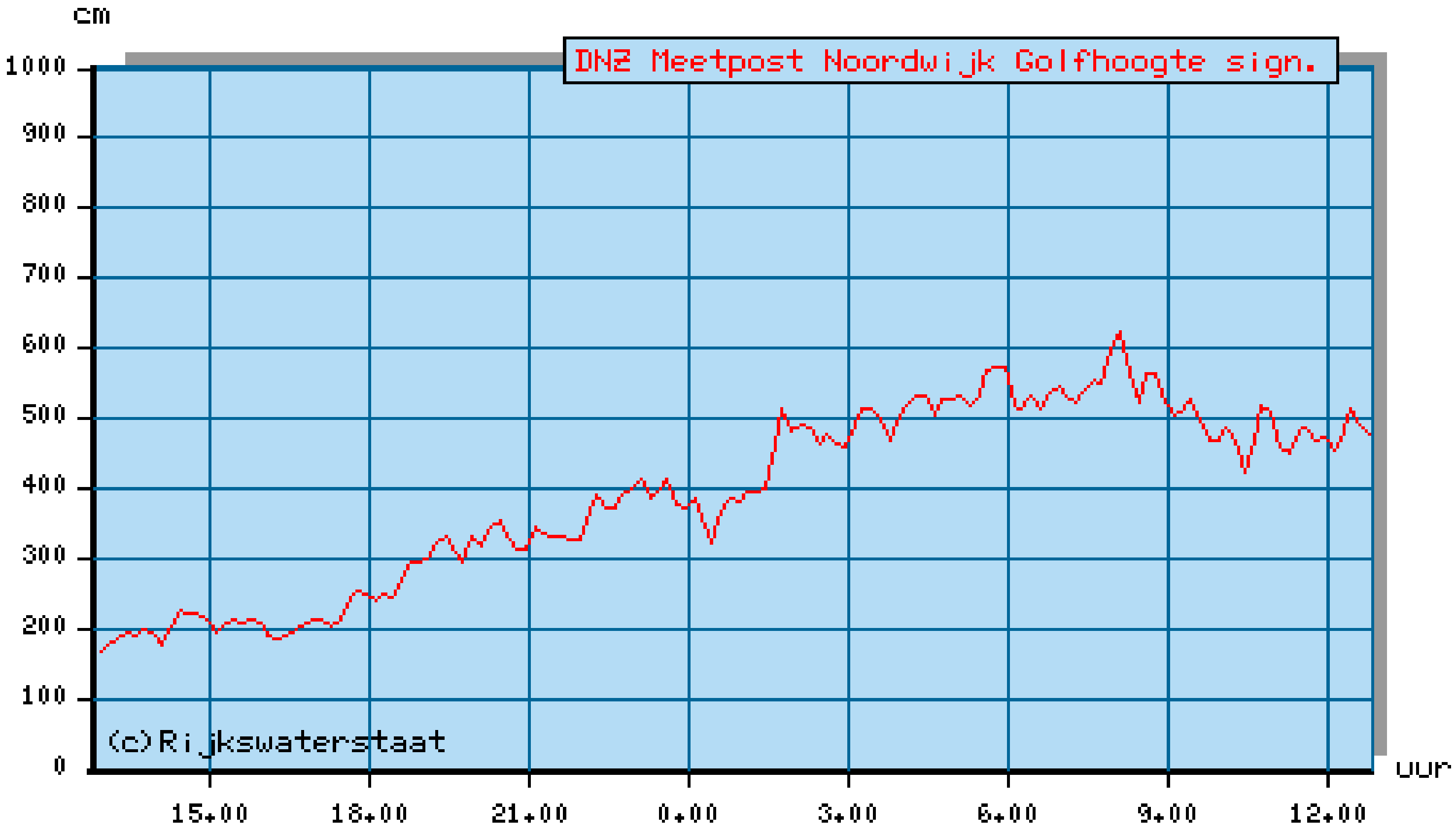


Water level



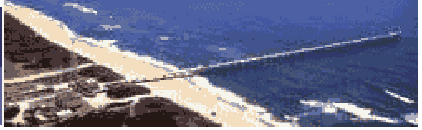
cm NAP
MSW Meetpost Noordwijk Astronomisch getij
DNZ Meetpost Noordwijk Waterstand
500
400
300
200
100
0
-100
-200
-300
-400
-500
(c)Rijkswaterstaat
uur
12+00
18+00
0+00
6+00
12+00
18+00
0+00
6+00

Significant wave height

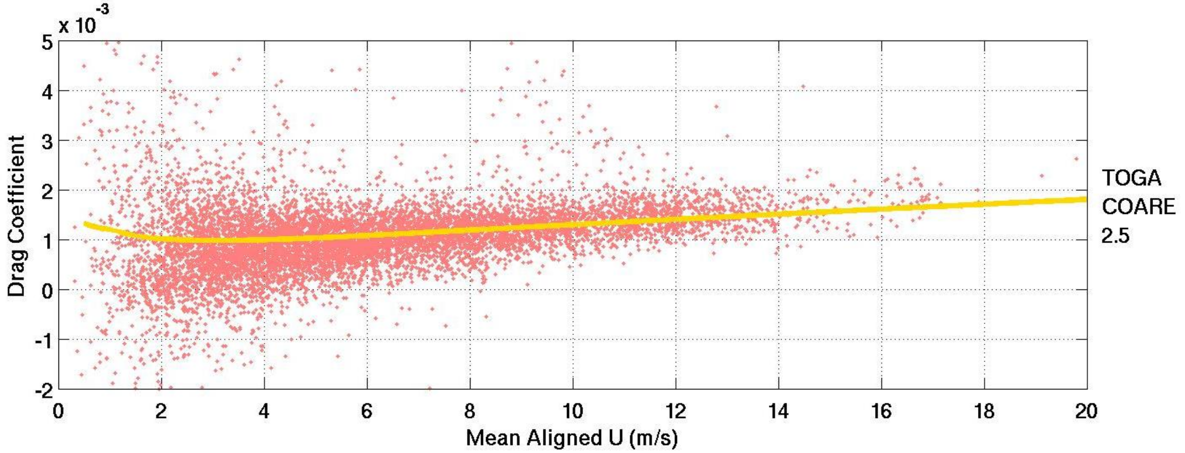


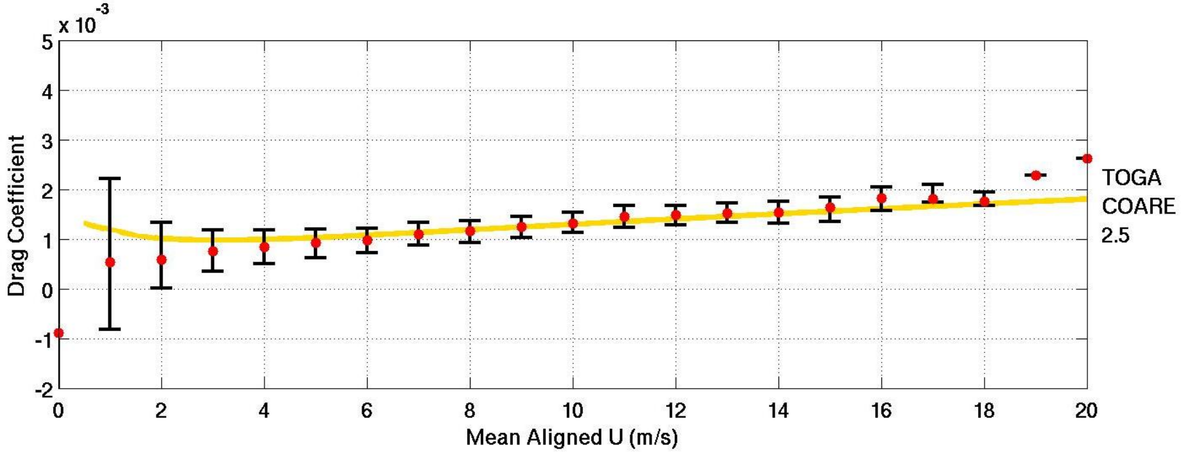
Storm surge

- Highest when wind is blowing towards coast
- Higher for small water depth
- Further increase with gently sloping bottom
- Additional wave setup
- Can lead to dune erosion

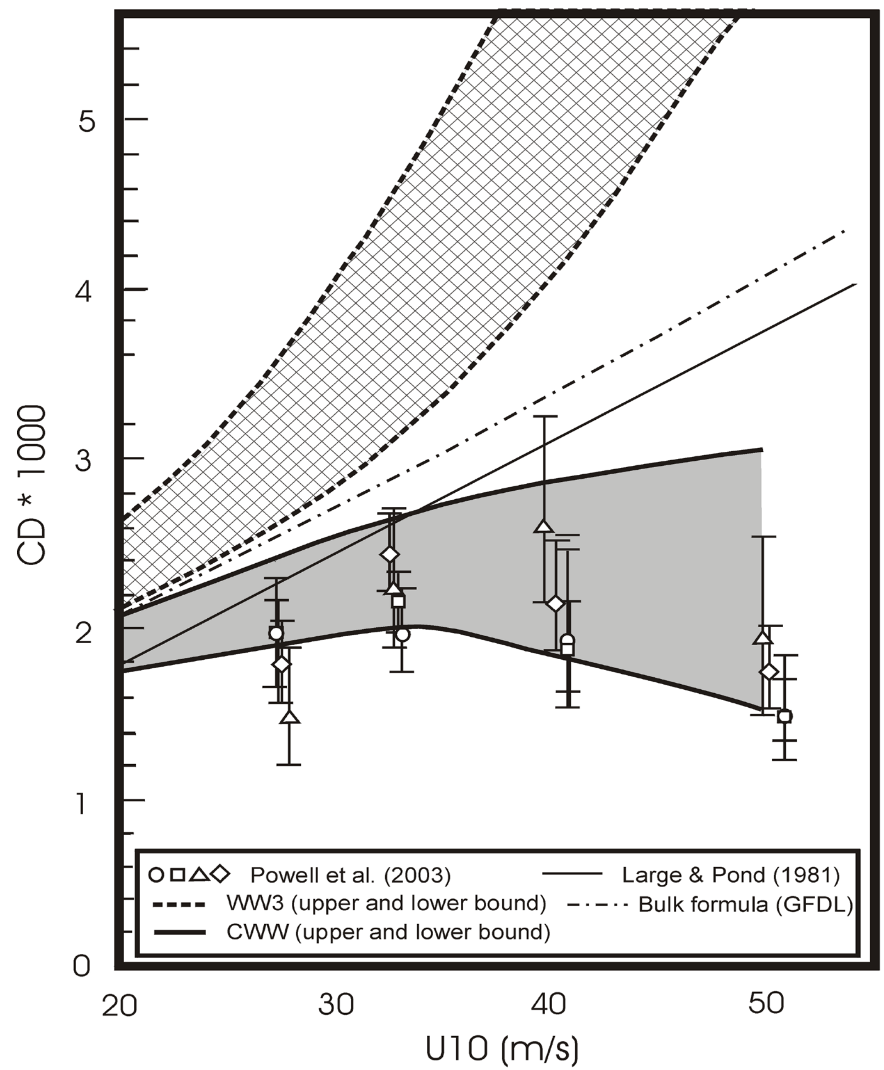


Drag coefficient vs. onshore winds (180-330º) LDEO (10 m) - October 2005 – May 2006





C_D for large wind speeds



Is this all there is to surges?

- No!
- Work by Maarten van Ormondt

Coastal Engineering

Volume 165, April 2021, 103839

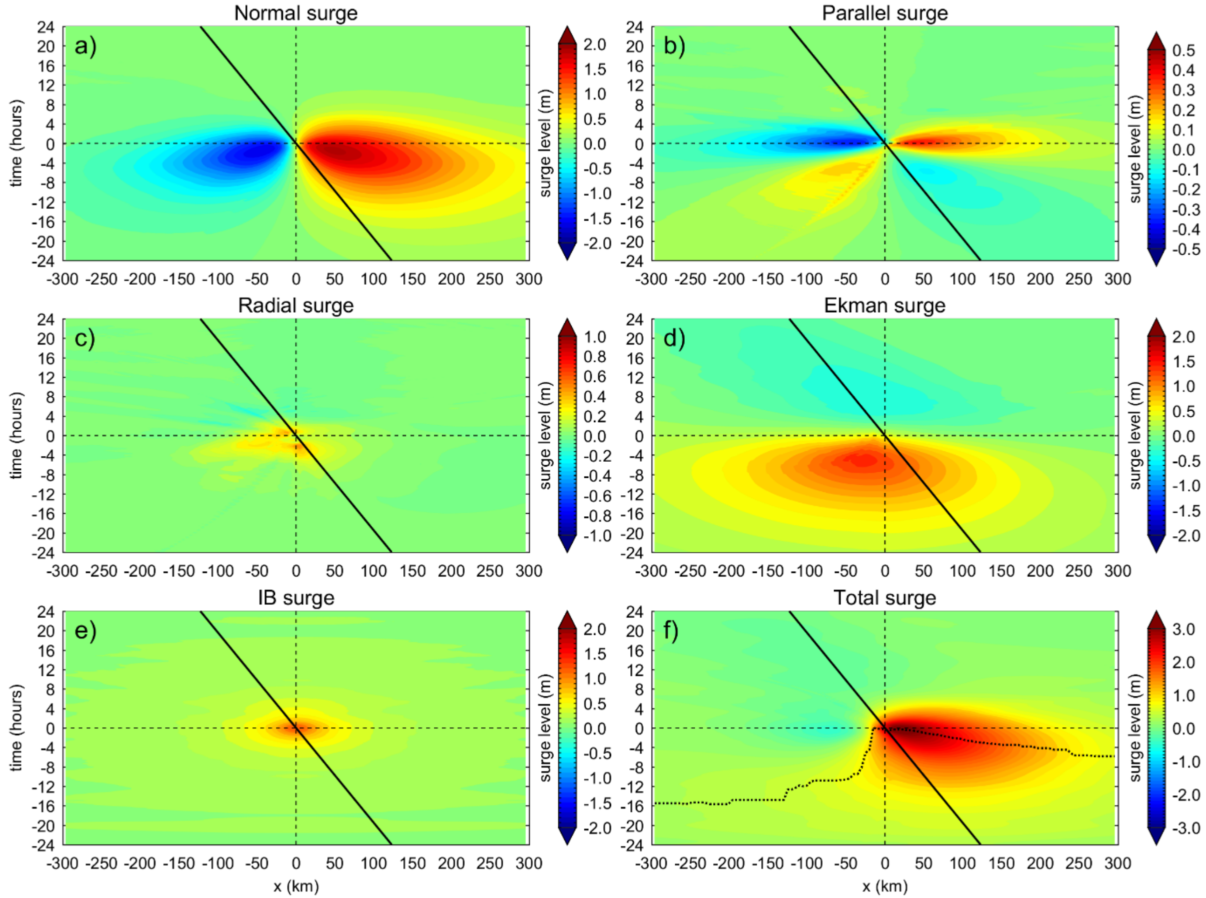
A semi-empirical method for computing storm surges on open coasts during tropical cyclones

Maarten van Ormondt [1], Ap van Dongeren [1], Dano Roelvink [2]

Show more

- **Normal surge S_{NOR} – resulting from wind-induced set-up due to shore-normal component of wind stress**
- Parallel surge S_{PAR} – resulting from wind-induced set-up due to shore-parallel gradients in the wind stress
- Radial surge S_{RAD} – resulting from wind set-up due to radial component of the wind field
- Ekman surge S_{EKM} – resulting from Coriolis force acting on alongshore wind-induced currents
- Inverse barometer surge S_{IBE} – resulting from gradients in atmospheric pressure

Example surge components



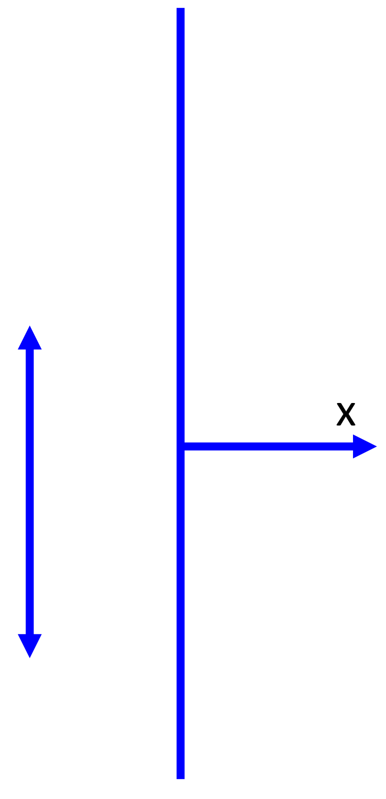
Time (hrs)

Distance from landfall (km)

Source: van Ormondt et al. (subm.)

Tidal currents on an open coast

- We assume straight coast
- Cross-shore velocities very small
- Gradients in velocity are small because wave length of tidal wave is very long



Tidal currents (open coast)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-f_{cor}v=\frac{\partial}{\partial y}D_h\frac{\partial u}{\partial y}+\frac{\tau_{sx}}{\rho h}-\frac{\tau_{bx}}{\rho h}-\frac{1}{\rho}\frac{\partial p_a}{\partial x}-g\frac{\partial \eta}{\partial x}+\frac{F_x}{\rho h}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+f_{cor}u=\frac{\partial}{\partial x}D_h\frac{\partial v}{\partial x}+\frac{\tau_{sy}}{\rho h}-\frac{\tau_{by}}{\rho h}-\frac{1}{\rho}\frac{\partial p_a}{\partial y}-g\frac{\partial \eta}{\partial y}+\frac{F_y}{\rho h}$$

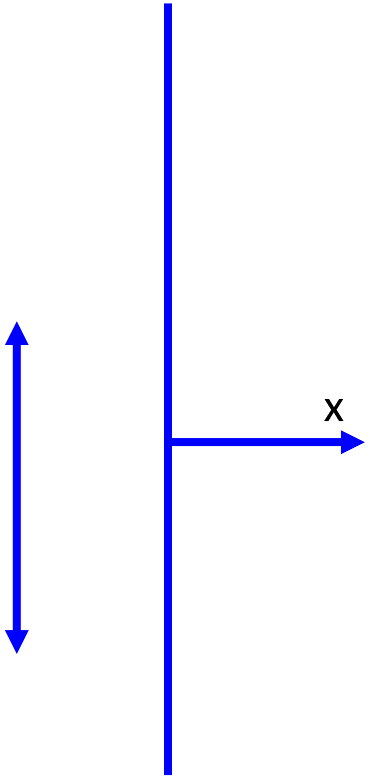
$$\frac{\partial h\overline{u}}{\partial x}+\frac{\partial h\overline{v}}{\partial y}+\frac{\partial \eta}{\partial t}=0$$

Neglect friction (relatively deep)

$$fv = g\frac{\partial \eta}{\partial x}$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + h\frac{\partial v}{\partial y} = 0$$



Assume sinusoidal tidal wave

$$\eta = \widehat{\eta}\cos(\omega t - ky),\ \ v = \widehat{v}\cos(\omega t - ky)$$

$$\widehat{v}\omega = gk\widehat{\eta} \quad \Big| \ \frac{\widehat{\eta}}{k}$$

$$\widehat{\eta}\omega = hk\widehat{v} \quad \Big| \ \frac{\widehat{v}}{k}$$

$$g\widehat{\eta}^2 - h\widehat{v}^2 = 0 \Rightarrow \widehat{v} = \widehat{\eta}\sqrt{\frac{g}{h}}$$

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Kelvin wave

$$fv = g\frac{\partial \eta}{\partial x} \qquad v = \eta\sqrt{\frac{g}{h}}$$

$$f\eta\sqrt{\frac{g}{h}} = g\frac{\partial \eta}{\partial x} \Rightarrow \frac{\partial \eta}{\partial x} = \frac{f}{\sqrt{gh}}\eta$$

$$\eta = \eta_0 \exp\left(\frac{f}{\sqrt{gh}}x\right)\cos\left(\omega t - ky\right)$$

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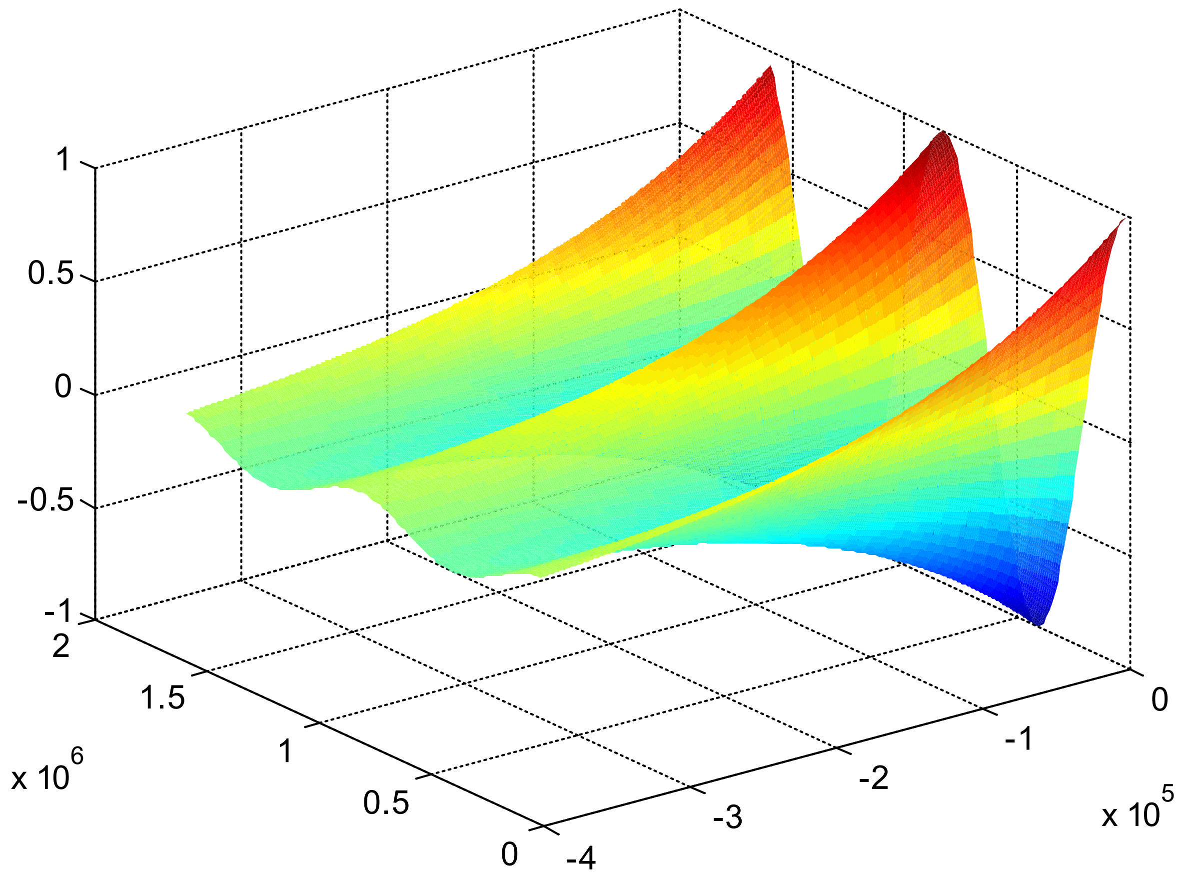
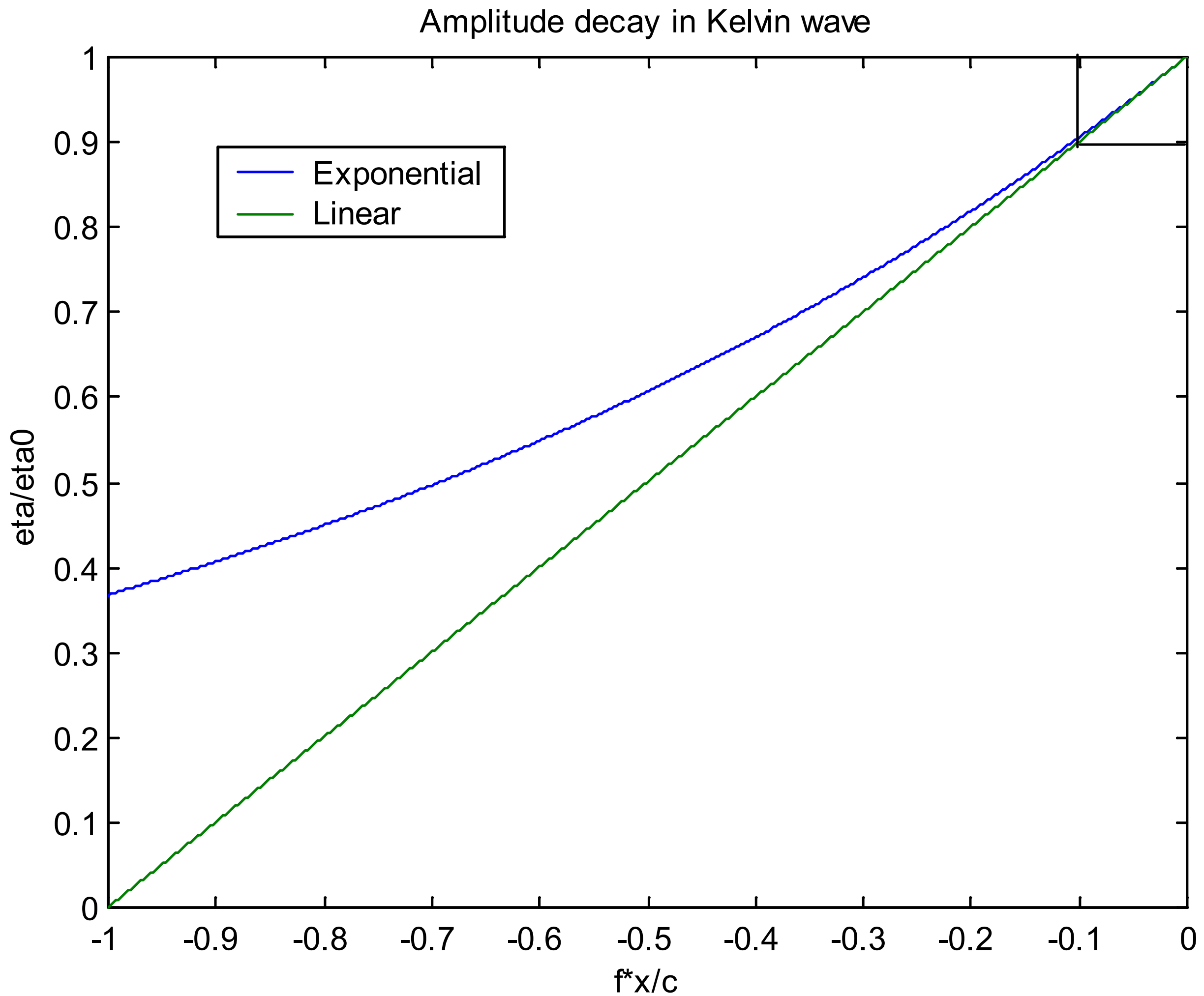
Kelvin wave



Kelvin wave



Amplitude decay in Kelvin wave
Exponential
Linear
eta/eta0
f*x/c
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0

Tidal alongshore current

- Important in shallow sea
- driven by alongshore pressure gradient
- Important terms: inertia, water level gradient, bottom friction

Longshore momentum balance

$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y} - \frac{\tau_{by}}{\rho h}$$

$$\frac{\tau_{by}}{\rho h} = \frac{\rho C_d \left|v\right| v}{\rho h} \approx \frac{\lambda}{h} v$$

$$\eta = \hat{\eta}\cos(\omega t - ky)$$

$$v = v_1 \cos(\omega t - ky) + v_2 \sin(\omega t - ky)$$

$$\frac{\partial \eta}{\partial y} = +k\hat{\eta}\sin(\omega t - ky)$$

$$\frac{\partial v}{\partial t} = -v_1 \omega \sin(\omega t - ky) + v_2 \omega \cos(\omega t - ky)$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y} - \frac{\lambda}{h}v \Rightarrow$$

$$-v_1\omega\sin(\omega t - kx) + v_2\omega\cos(\omega t - ky) =$$

$$-gk\hat{\eta}\sin(\omega t - ky) -$$

$$\left(\frac{\lambda}{h}v_1\cos(\omega t - ky) + \frac{\lambda}{h}v_2\sin(\omega t - ky)\right)$$

$$-v_1\omega\sin(\omega t-kx)=-gk\hat{\eta}\sin(\omega t-ky)-\frac{\lambda}{h}v_2\sin(\omega t-ky)$$

$$v_2\omega\cos(\omega t-ky)=-\frac{\lambda}{h}v_1\cos(\omega t-ky)\ \Rightarrow$$

$$v_2\omega=-\frac{\lambda}{h}v_1 \qquad v_2=-\frac{\lambda}{\omega h}v_1$$

$$-v_1\omega=-gk\hat{\eta}-\frac{\lambda}{h}v_2 \qquad v_1=\frac{gk\hat{\eta}}{\omega}-\frac{\lambda^2}{\omega^2h^2}v_1\Rightarrow$$

$$v_1\left(1+\frac{\lambda^2}{\omega^2h^2}\right)=\frac{gk\hat{\eta}}{\omega}\Rightarrow v_1=\frac{1}{\left(1+\left(\frac{\lambda}{\omega h}\right)^2\right)}\frac{gk\hat{\eta}}{\omega}$$

$$v=\frac{gk}{\omega}\frac{1}{1+\left(\frac{\lambda}{\omega h}\right)^2}\hat{\eta}\cos(\omega t-ky)\quad-\quad\frac{gk}{\omega}\frac{\frac{\omega h}{\lambda}}{1+\left(\frac{\omega h}{\lambda}\right)^2}\hat{\eta}\sin(\omega t-ky)\Rightarrow$$

$$v=\frac{gk\hat{\eta}}{\omega}\left(\frac{1}{1+\varphi^2}\cos(\omega t-ky)\quad-\quad\frac{\varphi}{1+\varphi^2}\sin(\omega t-ky)\right),\quad\varphi=\frac{\lambda}{\omega h}\Rightarrow$$

$$v=\frac{1}{\sqrt{\left(1+\varphi^2\right)}}\frac{gk\hat{\eta}}{\omega}\left(\cos(\omega t-ky+\arctan(\varphi))\ \right)$$

General solution

$$v = \frac{1}{\sqrt{\left(1+\varphi^2\right)}} \frac{gk\hat{\eta}}{\omega} \left(\cos(\omega t - ky + \arctan(\varphi)) \right)$$

$$\varphi = \frac{\lambda}{\omega h}$$

Shallow water solution

$$\omega / k = c$$

$$\varphi = \frac{\lambda}{\omega h}$$

$$v = -\frac{g}{c}\frac{1}{\varphi}\hat{\eta}\sin(\omega t - kx) = \frac{gkh}{\lambda}\hat{\eta}\sin(\omega t - kx)$$

Deep water

$$v = \frac{g}{c}\widehat{\eta}\cos(\omega t - kx)$$

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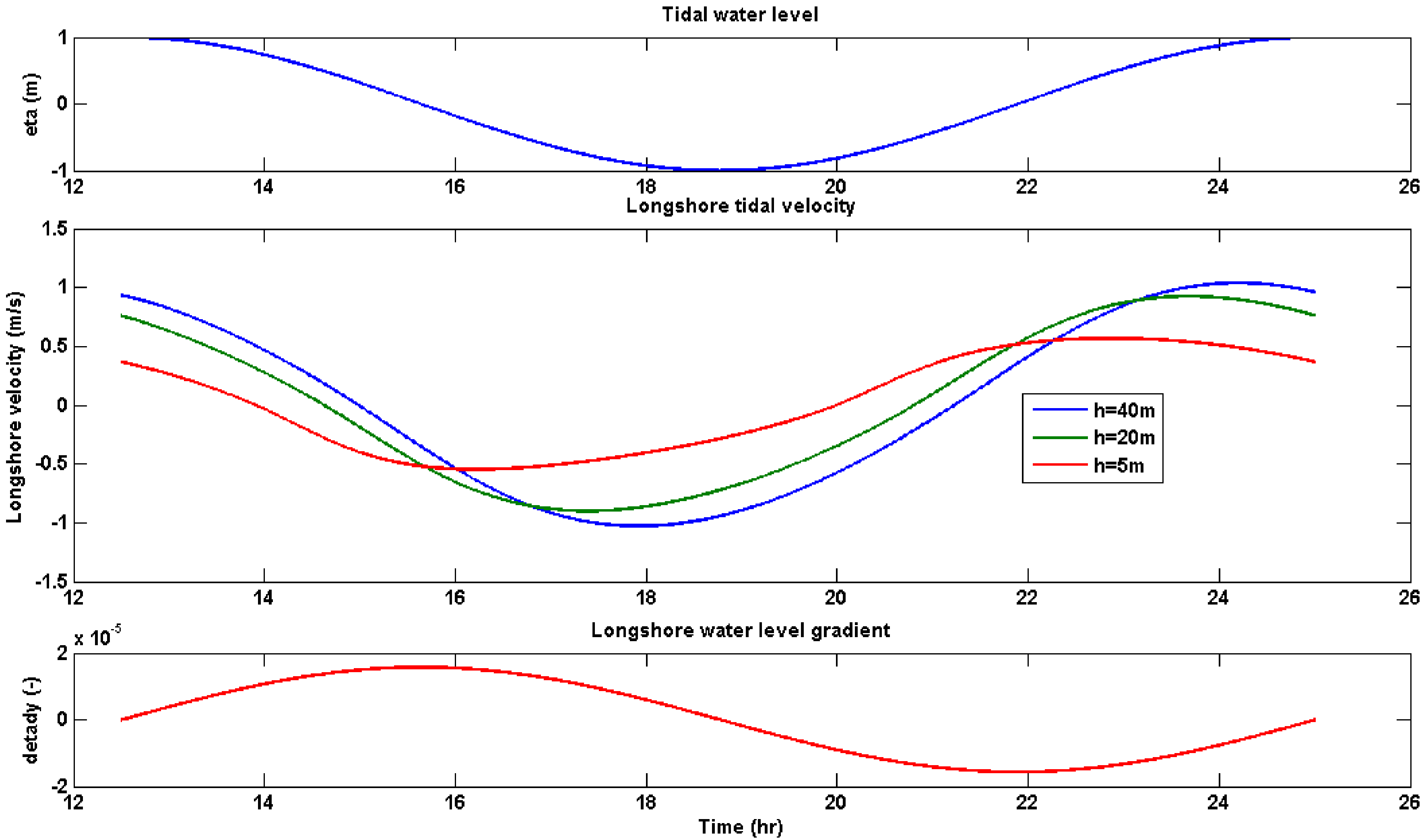
Tidal water level
eta (m)
1
0
-1
12
14
16
18
20
22
24
26
Longshore tidal velocity
Longshore velocity (m/s)
1.5
1
0.5
0
-0.5
-1
-1.5
h=40m
h=20m
h=5m
Longshore water level gradient
x 10^-5
detady (-)
2
0
-2
Time (hr)

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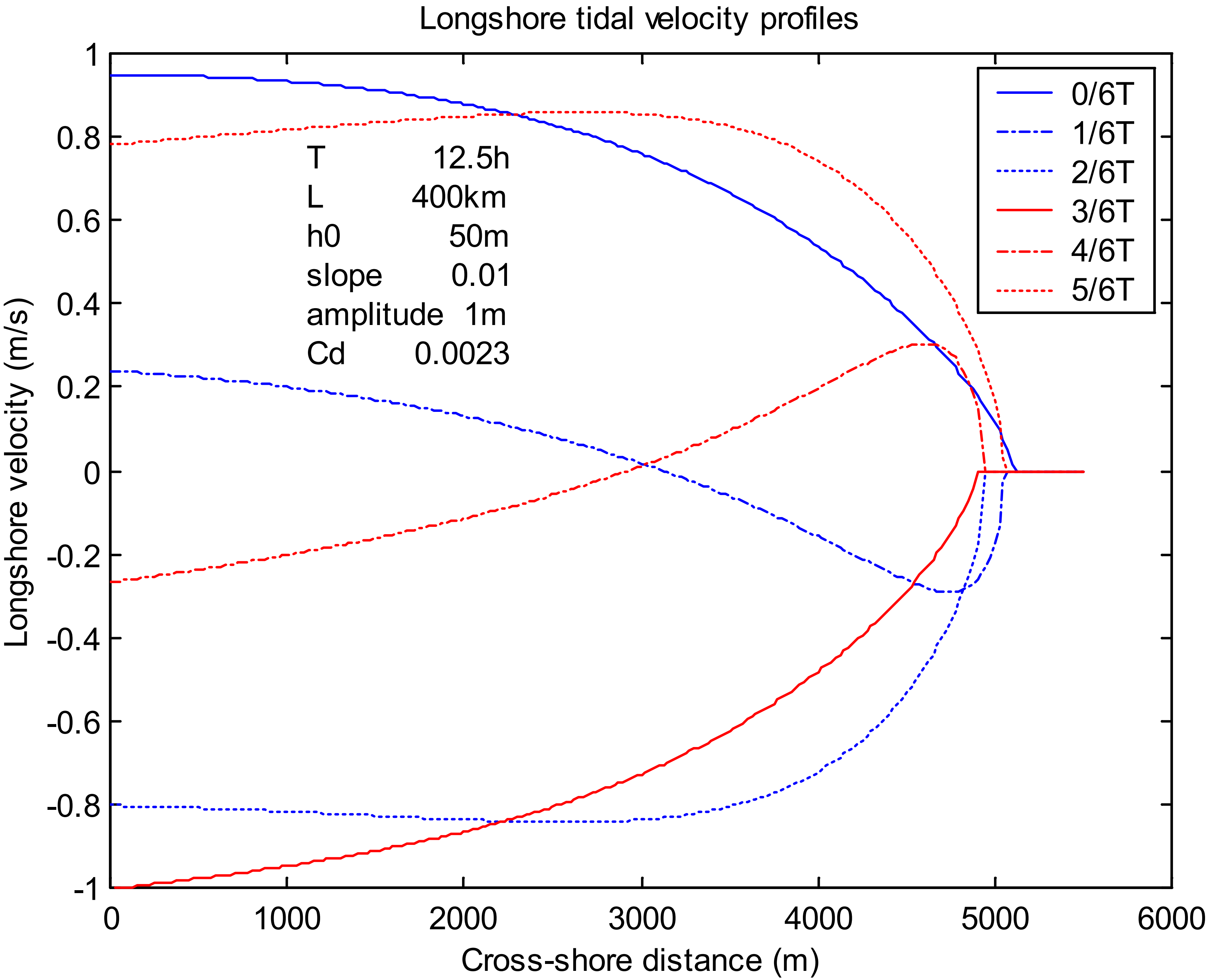
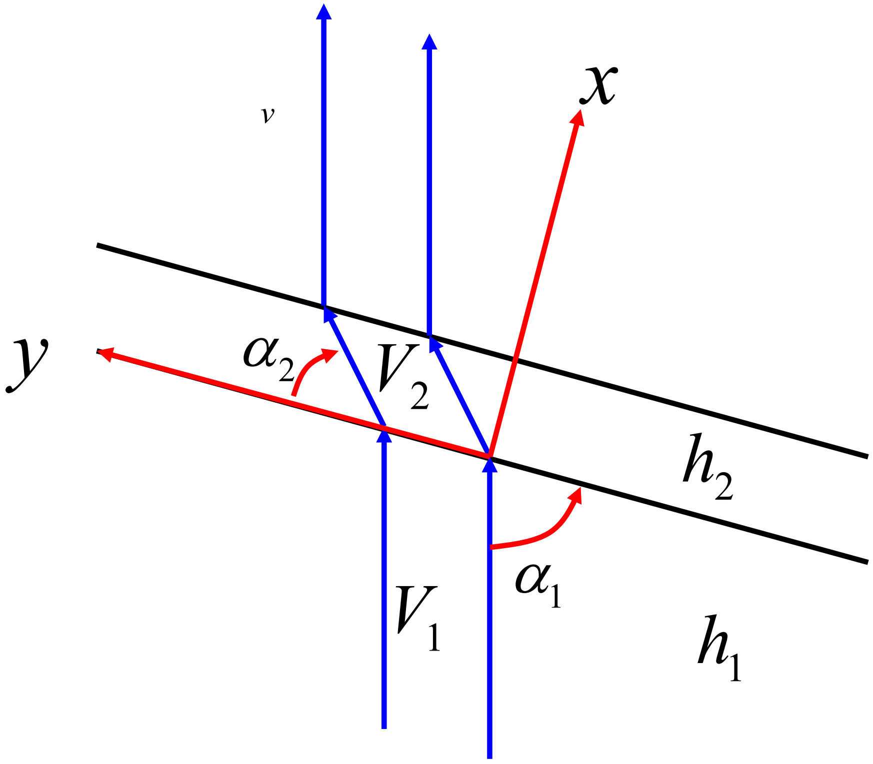

Longshore tidal velocity profiles
T 12.5h
L 400km
h0 50m
slope 0.01
amplitude 1m
Cd 0.0023
0/6T
1/6T
2/6T
3/6T
4/6T
5/6T
Longshore velocity (m/s)
Cross-shore distance (m)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1000
2000
3000
4000
5000
6000

Current pattern across a channel



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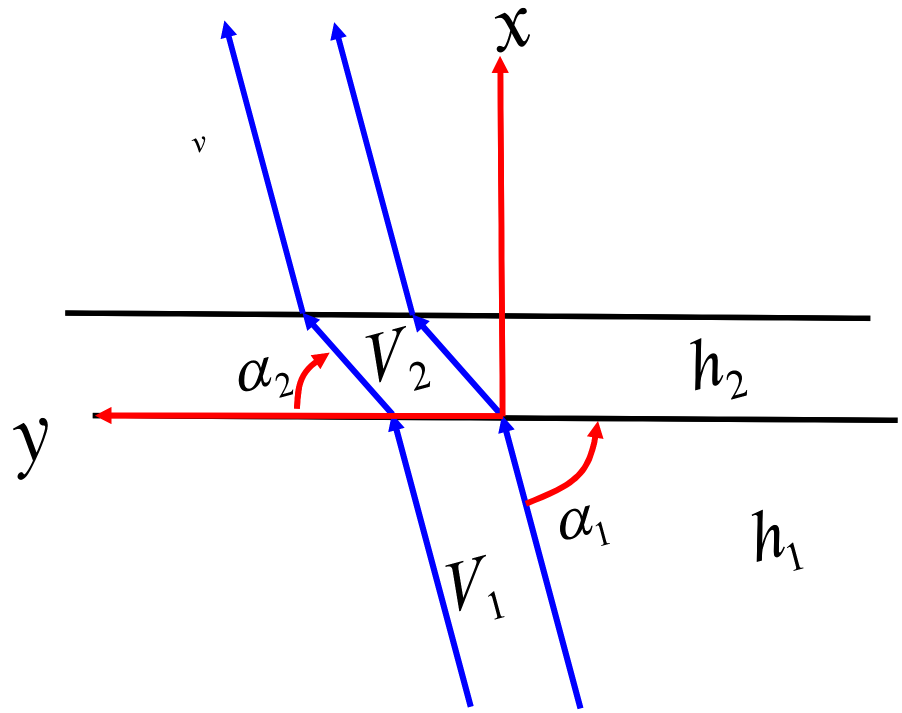
Approach

- Assume trench very long
- Water level gradient along trench constant
- All other gradients along trench zero
- Take x-axis across channel, y-axis along channel
- Solve u from continuity equation, v from momentum equation
- Page 49 in book

Current pattern across a channel



Continuity: cross-channel discharge is constant

$$u_1 = V_1 \sin(\alpha_1)$$

$$u_1 h_1 = u_2 h_2 \Rightarrow u_2 = u_1 \frac{h_1}{h_2}$$

Surface slope along channel is equal

$$|V_1|^2 = C^2 h_1 i_1 \Rightarrow i_1 = \frac{|V|_1^2}{C^2 h_1}$$

$$i_{y,2} = i_{y,1} = i_1 \cos(\alpha_1)$$

$$|V_2| v_2 = v_2 \sqrt{u_2^2 + v_2^2} = C^2 h_2 i_{y,2} \quad \Rightarrow$$

$$\Rightarrow \quad {v_2}^2 \left(u_2^2 + v_2^2\right) = C^4 h_2^2 i_{y,2}^2 \quad \Rightarrow$$

$$\Rightarrow \quad \left(v_2^2\right)^2 + u_2^2 {v_2}^2 - C^4 h_2^2 i_{y,2}^2 = 0 \quad \Rightarrow$$

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Solution

$$\left(v_2^2\right)^2 + u_2^2 v_2{}^2 - C^4 h_2^2 i_{y,2}^2 = 0 \quad \Rightarrow$$

$$v_2^2 = \frac{-u_2^2 + \sqrt{u_2^4 + 4C^4 h_2^2 i_{y,2}^2}}{2} \quad \Rightarrow$$

$$v_2 = \sqrt{\frac{-u_2^2 + \sqrt{u_2^4 + 4C^4 h_2^2 i_{y,2}^2}}{2}}$$

$$\alpha_2 = \arctan\left(\frac{u_2}{v_2}\right)$$

Assignment

- V_1=1 m/s;
- C=65 $m^{1/2}$/s;
- $alpha_1$=0(10)90 deg.
- h_1=10 m
- h_2=15 m

- Give V_2, $alpha_2$

Solution (see book p 52)