Coastal Hydrodynamics
Introduction and Wave Energy Balance

Prof. Dano Roelvink

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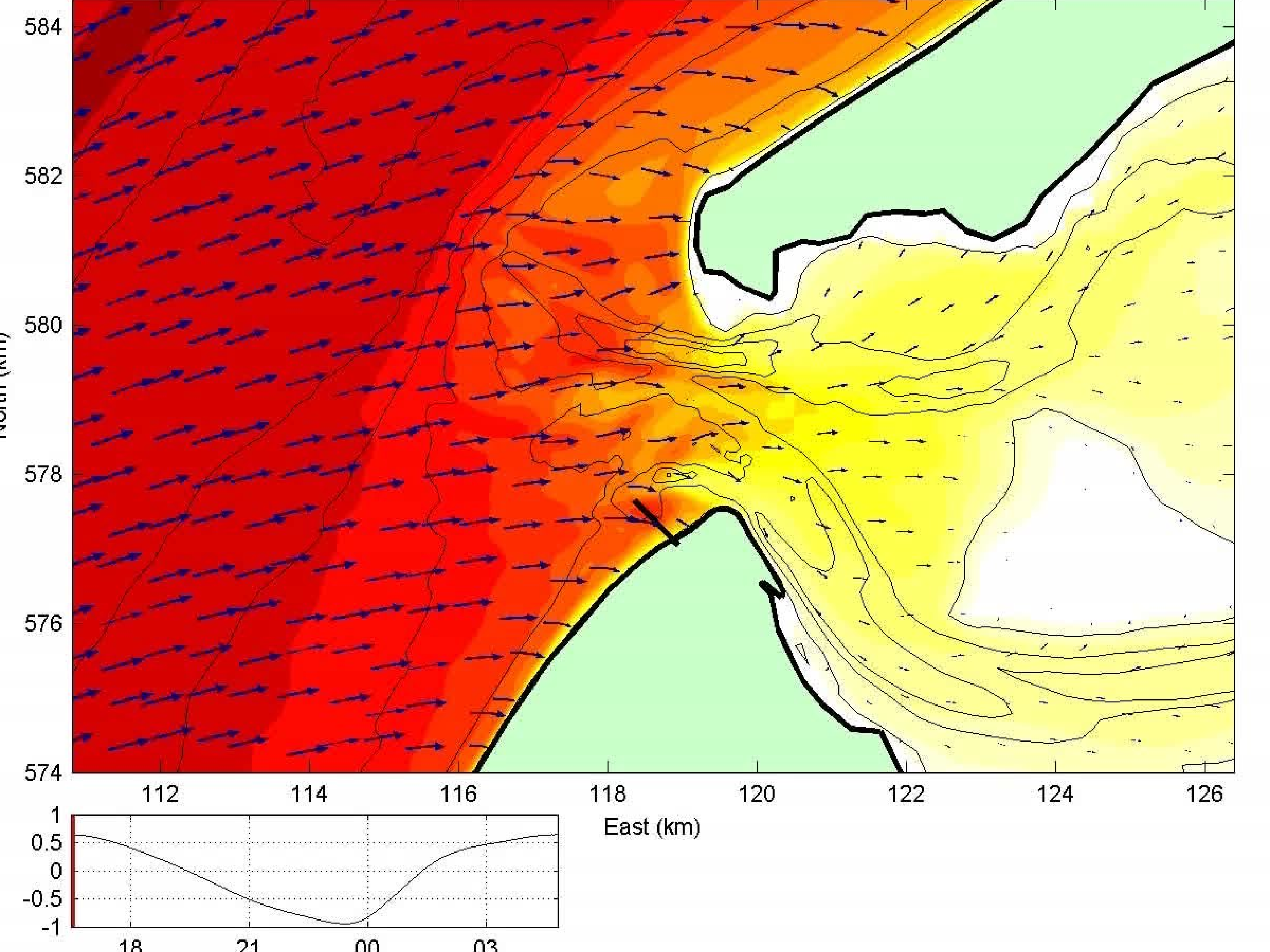


584
582
580
578
576
574
North (km)
112
114
116
118
120
122
124
126
East (km)
1
0.5
0
-0.5
-1
18
21
00
03

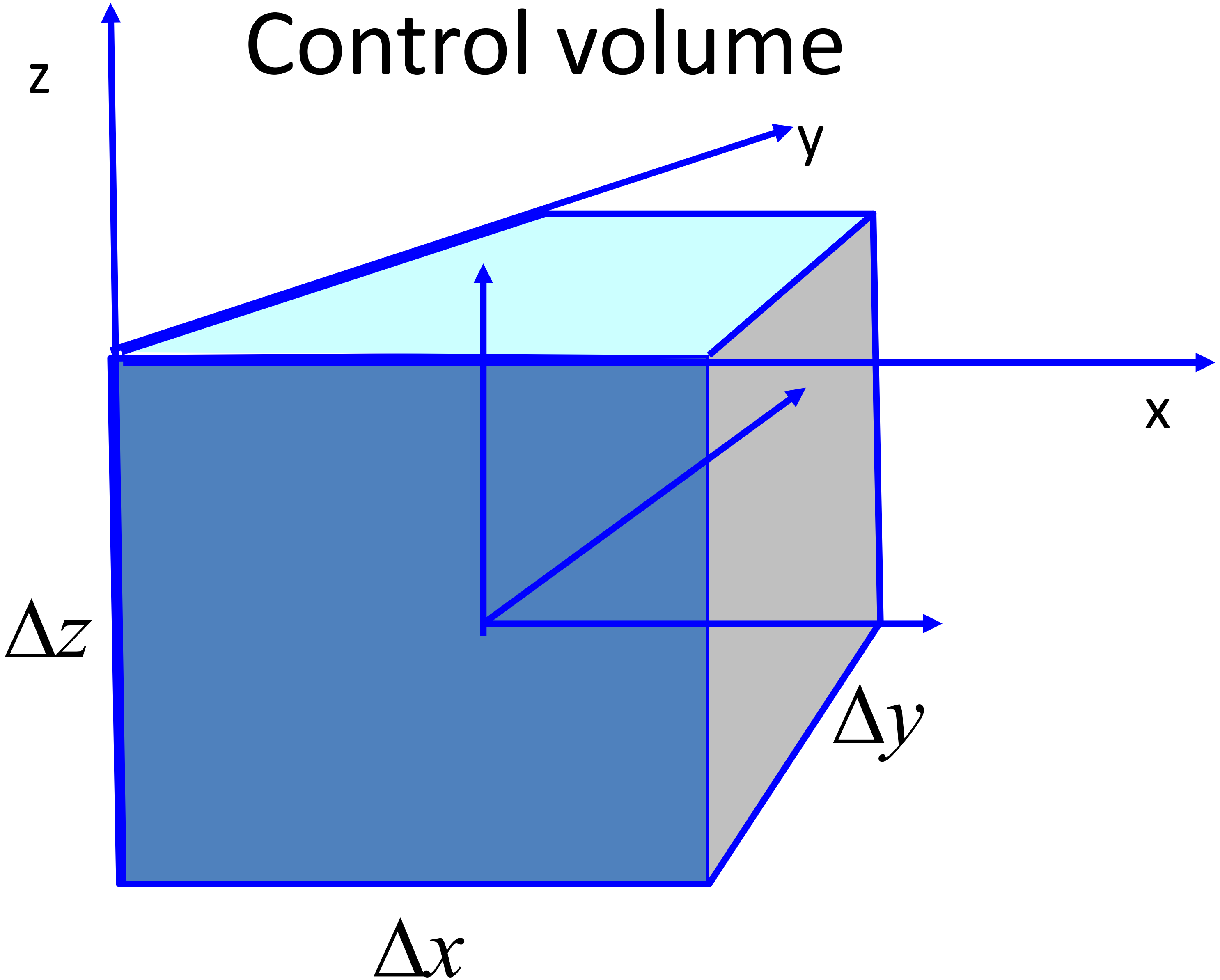
Currents and waves in coastal areas

- Shallow water equations
  - 3D
  - Depth-averaged
- Wave and roller energy balance
- Wave-driven currents
- Wind-driven currents
- Tidal currents
- Currents passing a channel

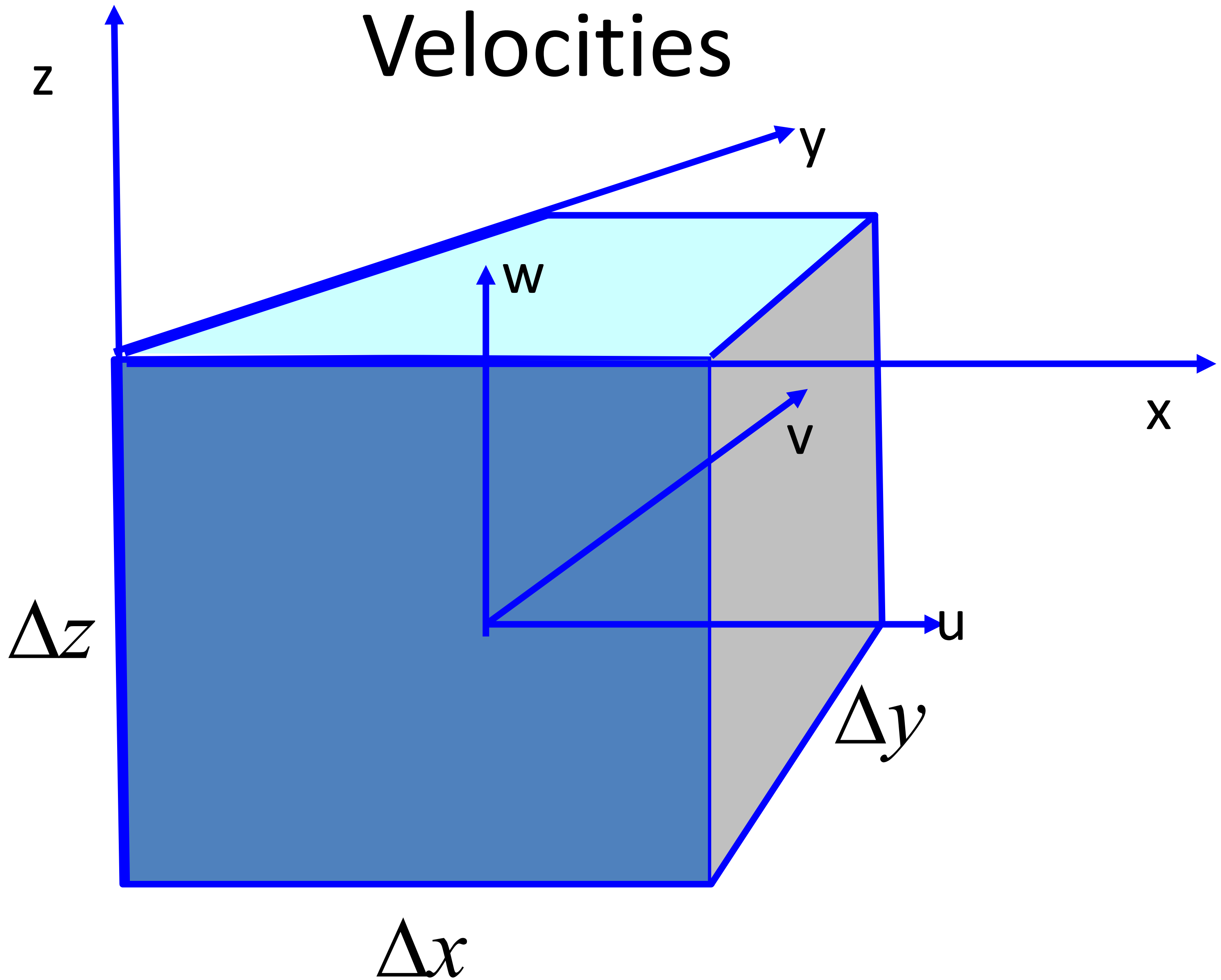
Objectives

- Understand background of basic equations
- Learn which terms are dominant in different situations
- Find simple solutions for schematised cases
- Form independent opinion on validity of complex model results

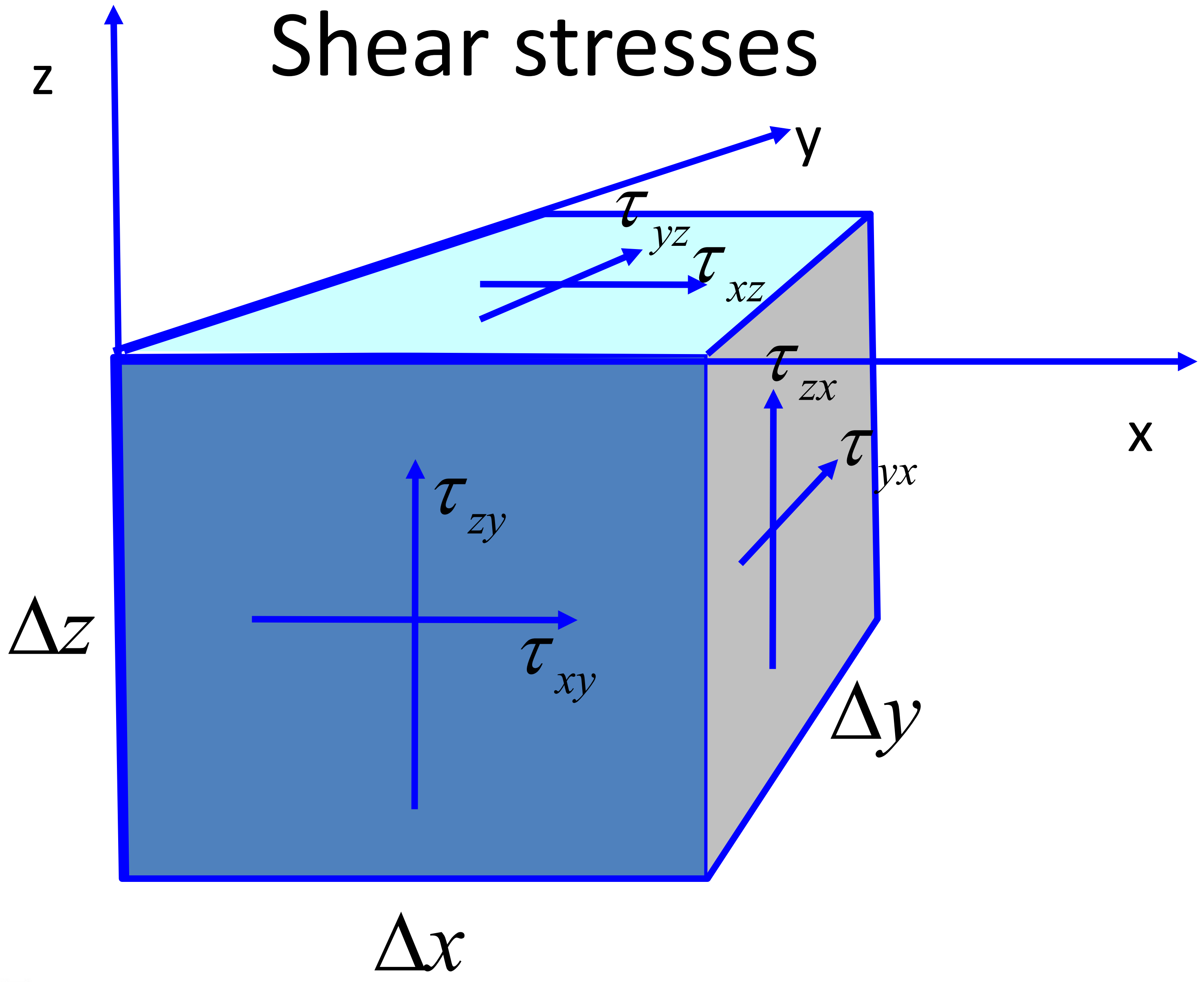
Control volume



Velocities



Shear stresses



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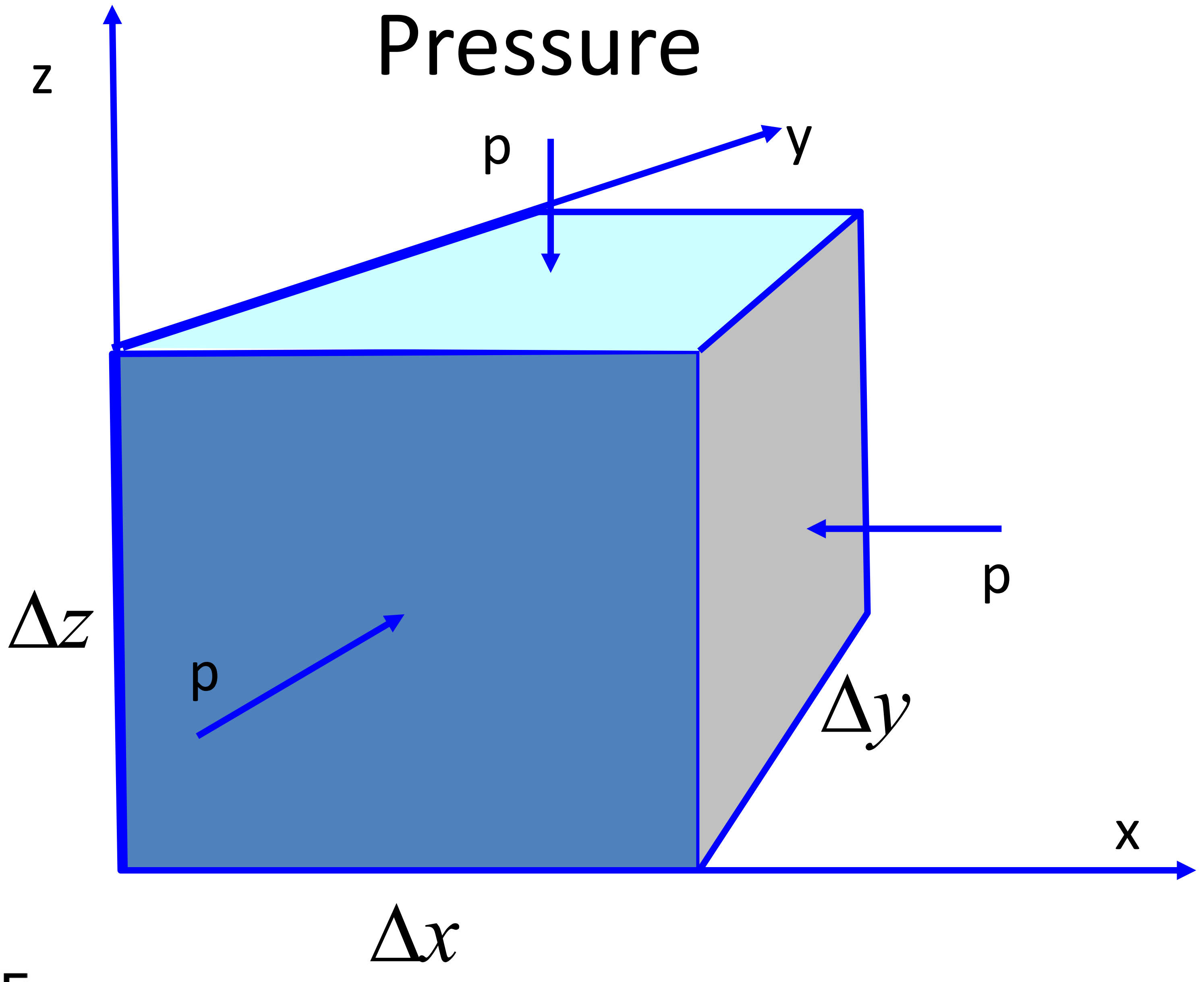
Pressure



Momentum balance

- *F=m.a*
- *F*= sum of stresses times area plus sum of pressure times area
- $m = \rho \Delta x \Delta y \Delta z$
- $a = \frac{du}{dt}$

Momentum balance (x)

$$\rho \Delta x \Delta y \Delta z \frac{du}{dt} = \Delta \tau_{xy} \Delta x \Delta z + \Delta \tau_{xz} \Delta x \Delta y - \Delta p \Delta y \Delta z \Rightarrow$$

$$\rho \frac{du}{dt} = \frac{\Delta \tau_{xy}}{\Delta y} + \frac{\Delta \tau_{xz}}{\Delta z} - \frac{\Delta p}{\Delta x} \quad \rightarrow$$

$$\rho \frac{du}{dt} = \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial p}{\partial x}$$

Momentum balance

$$\frac{du}{dt} - f_{cor} v = \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial p}{\partial x}\right)$$

$$\frac{dv}{dt} + f_{cor} u = \frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} - \frac{\partial p}{\partial y}\right)$$

$$\frac{dw}{dt} = \frac{1}{\rho}\left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} - \frac{\partial p}{\partial z}\right) - g$$

Mass balance: incompressible flow

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Averaging momentum balance over short timescales

- Turbulence
  - Reynolds stresses
  - Approximated by turbulent shear stresses

$$\tau_{xy} = \rho \nu_h \frac{\partial u}{\partial y},\ \tau_{yx} = \rho \nu_h \frac{\partial v}{\partial x},\ \tau_{xz} = \rho \nu_v \frac{\partial u}{\partial z},$$

$$\tau_{yz} = \rho \nu_v \frac{\partial v}{\partial z},\ \tau_{zx} = \rho \nu_h \frac{\partial w}{\partial x},\ \tau_{zy} = \rho \nu_h \frac{\partial w}{\partial y}$$

Averaging momentum balance over short timescales

- Waves
  - Radiation stresses
  - Approximated by linear theory
  - Details in Short Waves lectures

Shallow water approximation

- Horizontal scales >> vertical scales
- Vertical velocities << horizontal velocities
- Neglect vertical acceleration

$$\frac{dw}{dt} = \frac{1}{\rho}\left(\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} - \frac{\partial p}{\partial z}\right) - g \Rightarrow$$

$$\frac{\partial p}{\partial z} = -\rho g$$

Hydrostatic pressure

- Inhomogeneous (density not constant):

$$p = p_a + g\int_z^\eta \rho dz$$

- Homogeneous (density constant):

$$p = p_a + \rho g(\eta - z)$$

Shallow Water Equations (3D)

$$\frac{du}{dt} - f_{cor}v = \frac{\partial}{\partial y}\left(\nu_h \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\nu_v \frac{\partial u}{\partial z}\right) - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{w_x}{\rho}$$

$$\frac{dv}{dt} + f_{cor}u = \frac{\partial}{\partial x}\left(\nu_h \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\nu_v \frac{\partial v}{\partial z}\right) - \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{w_y}{\rho}$$

Acceleration

Coriolis

Horizontal diffusion

Vertical diffusion

Horizontal pressure gradient

Wind, wave forcing

Boundary conditions

$$w = 0$$

$$\rho \nu_v \frac{\partial u}{\partial z} = \tau_{bx}$$

$$\rho \nu_v \frac{\partial v}{\partial z} = \tau_{by}$$

Bottom (z=-d)

$$w = \frac{\partial \eta}{\partial t}$$

$$\rho \nu_v \frac{\partial u}{\partial z} = \tau_{sx}$$

$$\rho \nu_v \frac{\partial v}{\partial z} = \tau_{sy}$$

Surface ($z = \eta$)

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From moving to fixed frame of reference

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

Shallow Water Equations (3D)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-f_{cor}v=\frac{\partial}{\partial y}\left(\nu_h\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu_v\frac{\partial u}{\partial z}\right)-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{w_x}{\rho}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+f_{cor}u=\frac{\partial}{\partial x}\left(\nu_h\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial z}\left(\nu_v\frac{\partial v}{\partial z}\right)-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{w_y}{\rho}$$

$$\frac{\partial h\overline{u}}{\partial x}+\frac{\partial h\overline{v}}{\partial x}+\frac{\partial \eta}{\partial t}=0$$

$$p=p_a+\int_z^\eta \rho g dz$$

Basis for Delft3D, POM, ROMS, Mike 3

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Depth-averaged momentum balance

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} - f_{cor}\overline{v} = \frac{\partial}{\partial y} D_h \frac{\partial \overline{u}}{\partial y} + \frac{\tau_{sx}}{\rho h} - \frac{\tau_{bx}}{\rho h} - \frac{1}{\rho}\frac{\partial p_a}{\partial x} - g\frac{\partial \eta}{\partial x} + \frac{F_x}{\rho h}$$

$$\frac{\partial \overline{v}}{\partial t} + \overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} + f_{cor}\overline{u} = \frac{\partial}{\partial x} D_h \frac{\partial \overline{v}}{\partial x} + \frac{\tau_{sy}}{\rho h} - \frac{\tau_{by}}{\rho h} - \frac{1}{\rho}\frac{\partial p_a}{\partial y} - g\frac{\partial \eta}{\partial y} + \frac{F_y}{\rho h}$$

Shallow water equations (2DH)

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} - f_{cor}\overline{v} = \frac{\partial}{\partial y} D_h \frac{\partial \overline{u}}{\partial y} + \frac{\tau_{sx}}{\rho h} - \frac{\tau_{bx}}{\rho h} - \frac{1}{\rho}\frac{\partial p_a}{\partial x} - g\frac{\partial \eta}{\partial x} + \frac{F_x}{\rho h}$$

$$\frac{\partial \overline{v}}{\partial t} + \overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} + f_{cor}\overline{u} = \frac{\partial}{\partial x} D_h \frac{\partial \overline{v}}{\partial x} + \frac{\tau_{sy}}{\rho h} - \frac{\tau_{by}}{\rho h} - \frac{1}{\rho}\frac{\partial p_a}{\partial y} - g\frac{\partial \eta}{\partial y} + \frac{F_y}{\rho h}$$

Wave forcing

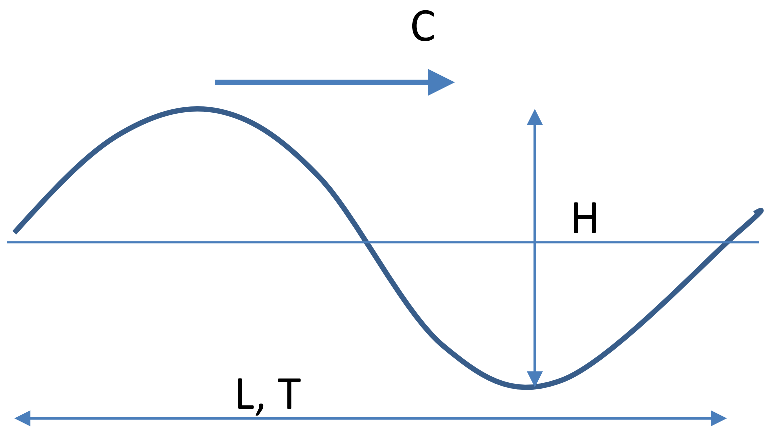
$$\frac{\partial h\overline{u}}{\partial x} + \frac{\partial h\overline{v}}{\partial y} + \frac{\partial \eta}{\partial t} = 0$$

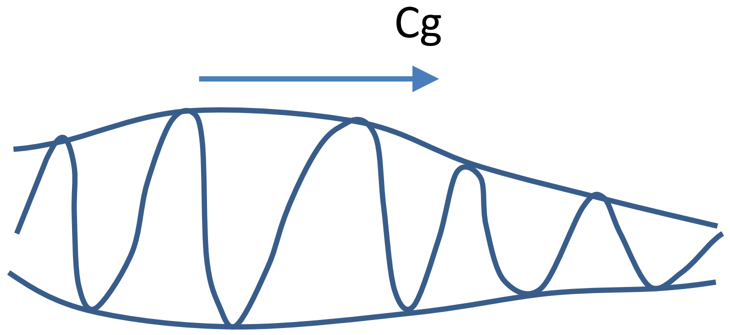
Basis for Delft3D, XBeach, Mike21

Waves and wave forcing

- Focus on nearshore
- Propagation and dissipation
- Waves driving current

Wave properties





Waves and wave forcing

- Wave energy balance
- Dispersion relation
- Wave celerity and group velocity
- Snel’s Law
- Shoaling and refraction
- Wave breaking
- Dissipation
- Solving 1D energy balance
- Radiation stresses and wave forces

Wave energy balance

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}\left(EC_g \cos\left(\vartheta\right)\right) + \frac{\partial}{\partial y}\left(EC_g \sin\left(\vartheta\right)\right) = -D_w$$

$$E = \rho g < \eta^2 > = \frac{1}{2}\rho g a^2 = \frac{1}{8}\rho g H_{rms}^2$$

Wave energy

Group velocity

Wave angle

Wave dissipation

Dispersion relation

- Relation between wave period T and wave length L for given water depth

$$\omega^2 = gk\tanh(kh)$$

$$\omega = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{L} \qquad C = \frac{L}{T} = \frac{\omega}{k}$$

$$C_g = \frac{d\omega}{dk} = nC = \left(\frac{1}{2} + \frac{kh}{\sinh(2kh)}\right)C$$

Deep water

$$\tanh(kh) \rightarrow 1 \qquad \sinh(kh) \rightarrow \infty$$

$$\omega^2 = gk \Rightarrow k = \frac{\omega^2}{g}$$

$$\omega = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{L} \qquad C = \frac{L}{T} = \frac{\omega}{k} = \frac{g}{\omega} = \frac{gT}{2\pi}$$

$$C_g = \frac{d\omega}{dk} = nC = \frac{1}{2}C$$

Shallow water

$$\tanh(kh) \to kh \qquad \sinh(kh) \to kh$$

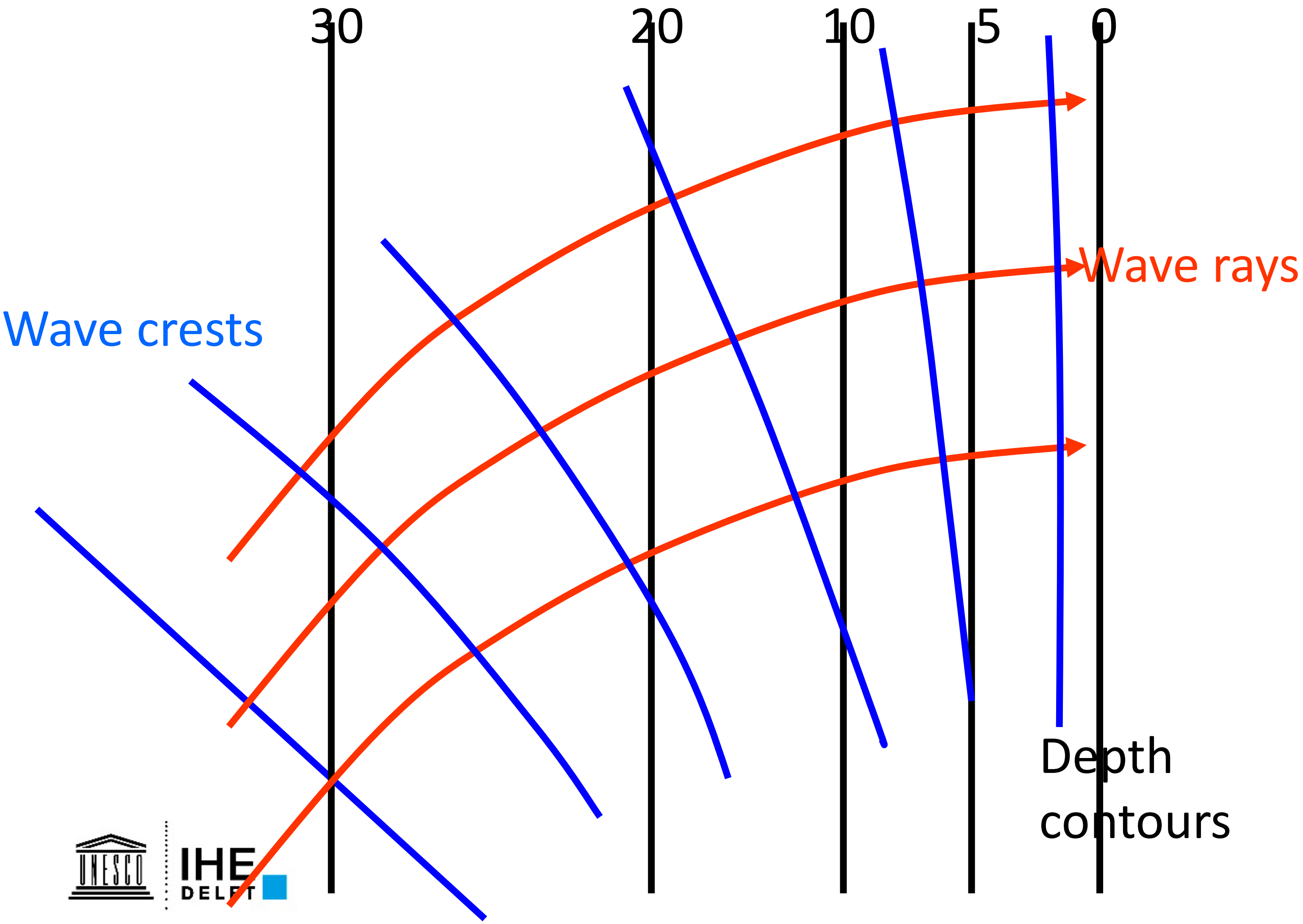
$$\omega^2 = gk\,(kh) \Rightarrow \omega / k = \sqrt{gh}$$

$$\omega = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{L} \qquad C = \frac{L}{T} = \frac{\omega}{k} = \sqrt{gh}$$

$$C_g = \frac{d\omega}{dk} = nC = \left(\frac{1}{2} + \frac{kh}{\sinh(2kh)} \right) C = \sqrt{gh}$$

Wave refraction



Snel's Law

- Valid for straight contour lines
- Relates local wave angle to deep water wave angle

$$\frac{\sin \vartheta}{c} = \frac{\sin \vartheta_0}{c_0}$$

Wave breaking

- Complex process
- Various approximations, e.g.
  - Battjes and Janssen, 1978
  - Thornton and Guza, 1983
  - Roelvink, 1993
  - Baldock, 1998

Baldock model

$$D_w = \frac{1}{4}\alpha\rho g\, f_p\, e^{-\left(\frac{H_{\max}}{H_{rms}}\right)^2}\left(H_{\max}^2 + H_{rms}^2\right)$$

Coefficient
~1

Coefficient
~0.6-0.8

$$H_{\max} = \frac{0.88}{k}\tanh\left(\frac{\gamma k h}{0.88}\right)$$

1D Wave energy balance

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}\left(EC_g \cos(\vartheta)\right) + \frac{\partial}{\partial y}\left(EC_g \sin(\vartheta)\right) = -D_w$$

Wave energy

Group velocity

Wave angle

Wave dissipation

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Solving 1D wave energy balance

- Outside surf zone:
  - Wave energy flux is constant
  - Group velocity follows from dispersion relation
  - Wave angle follows from Snel's Law

$$\frac{\partial}{\partial x}\left(EC_g \cos\left(\vartheta\right)\right) = 0 \Rightarrow$$

$$EC_g \cos\left(\vartheta\right) = E_0 C_{g0} \cos\left(\vartheta_0\right)$$

Shoaling and refraction

- Shoaling is change in wave height due to change in group velocity
- Mostly increasing towards shore
- Refraction is bending of wave rays towards shore, leads to decrease of wave height because energy is spread over wider area

$$\frac{E}{E_0} = \frac{C_{g0}}{C_g}\frac{\cos\vartheta_0}{\cos\vartheta} \Rightarrow \frac{H}{H_0} = \sqrt{\frac{C_{g0}}{C_g}}\sqrt{\frac{\cos\vartheta_0}{\cos\vartheta}}$$

Example

- Deep water conditions:
  - Wave height 1 m
  - Wave period 10 s
  - Wave direction 30 deg. w.r.t. normal
- Shallow water:
  - Water depth 2 m
  - Wave direction?
  - Wave height?

Solving 1D wave energy balance

- Inside surf zone:
  - Group velocity follows from dispersion relation
  - Wave angle follows from Snel's Law
  - Dissipation follows from e.g. Baldock, relating wave dissipation to wave energy and water depth
  - Solve *E* numerically, starting from known value outside breaker zone

$$\frac{\partial}{\partial x}\left(EC_g \cos\left(\vartheta\right)\right) = -D_w(E,h)$$

Wave forces

$$F_x = -\left(\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} \right)$$

$$F_y = -\left(\frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \right)$$

Radiation stresses

$$S_{xx} = \left(n\cos^2\vartheta + n - \frac{1}{2} \right) E$$

$$S_{xy} = S_{yx} = \left(n\cos\vartheta\sin\vartheta \right) E$$

$$S_{yy} = \left(n\sin^2\vartheta + n - \frac{1}{2} \right) E$$

$$n = \frac{C_g}{C}$$

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Wave forces

- Follow from radiation stress gradients
- Radiation stresses are function of wave energy, wave direction and ratio wave celerity to group velocity
- In 1D case we can compute all these easily

1D case
(cross-shore profile model)

- Assume straight and parallel contour lines
- All terms $\partial / \partial y$ vanish
- Cross-shore wave forces and longshore wave forces can be easily computed from the solution of the 1D wave energy balance
- There is a weak feedback through depth to the wave energy balance

Cross-shore wave forces

$$F_x = -\left(\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}\right) =$$

$$= -\frac{\partial}{\partial x}\left[\left(n\cos^2\vartheta + n - \frac{1}{2}\right)E\right]$$

Cross-shore momentum balance

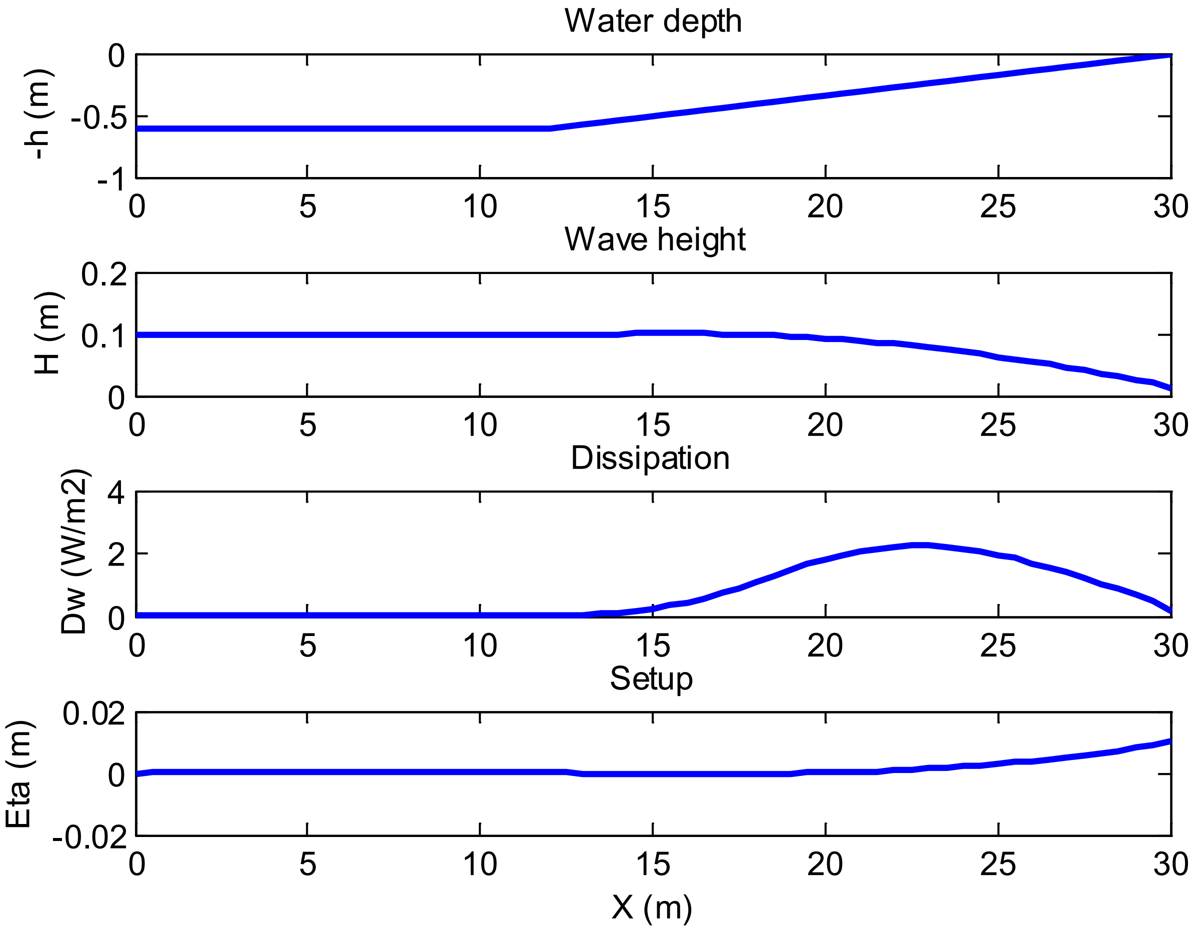
- Perpendicularly incident waves (flume):

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} - f_{cor}\overline{v} = \frac{\partial}{\partial y} D_h \frac{\partial \overline{u}}{\partial y} + \frac{\tau_{sx}}{\rho h} - \frac{\tau_{bx}}{\rho h} - \frac{1}{\rho}\frac{\partial p_a}{\partial x} - g\frac{\partial \eta}{\partial x} + \frac{F_x}{\rho h}$$

$$g\frac{\partial \eta}{\partial x} = \frac{F_x}{\rho h} \Rightarrow$$

$$\Rightarrow \frac{\partial \eta}{\partial x} = \frac{1}{\rho g h}\frac{\partial}{\partial x}\left[\left(2n - \frac{1}{2}\right)E\right]$$

Example for flume test



Longshore wave forces

$$F_y = -\left(\frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \right) =$$

$$= -\frac{\partial}{\partial x}\left[\left(n \cos\vartheta \sin\vartheta \right) E \right]$$

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Longshore wave forces

$$F_y = -\frac{\partial S_{xy}}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{C_g}{C}\left(E\cos\left(\vartheta\right)\sin\left(\vartheta\right)\right)\right]$$

$$= -\frac{\partial}{\partial x}\left[\frac{\sin\left(\vartheta\right)}{C} E C_g \cos\left(\vartheta\right)\right]$$

Longshore wave forces

=0

(Snel's Law)

$$F_y = -EC_g \cos(\vartheta) \frac{\partial}{\partial x}\left[\frac{\sin(\vartheta)}{C}\right]$$

$$-\frac{\sin(\vartheta)}{C}\frac{\partial}{\partial x}\left(EC_g \cos(\vartheta)\right) \Rightarrow$$

$$F_y = \sin(\vartheta)\frac{D_w}{C}$$

Wave-driven current

Neglect for now

$$\frac{\partial \overline{v}}{\partial t} + \overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} + f_{cor}\overline{u} = \frac{\partial}{\partial x} D_h \frac{\partial \overline{v}}{\partial x} + \frac{\tau_{sy}}{\rho h} - \frac{\tau_{by}}{\rho h} - \frac{1}{\rho}\frac{\partial p_a}{\partial y} - g\frac{\partial \eta}{\partial y} + \frac{F_y}{\rho h}$$

$$\tau_{by} = F_y \Rightarrow \rho C_f u_{rms} v = \sin(\vartheta)\frac{D_w}{C} \Rightarrow$$

$$\Rightarrow \rho C_f u_{rms} v = \sin(\vartheta)\frac{D_w}{C}$$

$$v = \frac{\sin(\vartheta)}{\rho C_f u_{rms}}\frac{D_w}{C}$$

Waves and wave driven currents

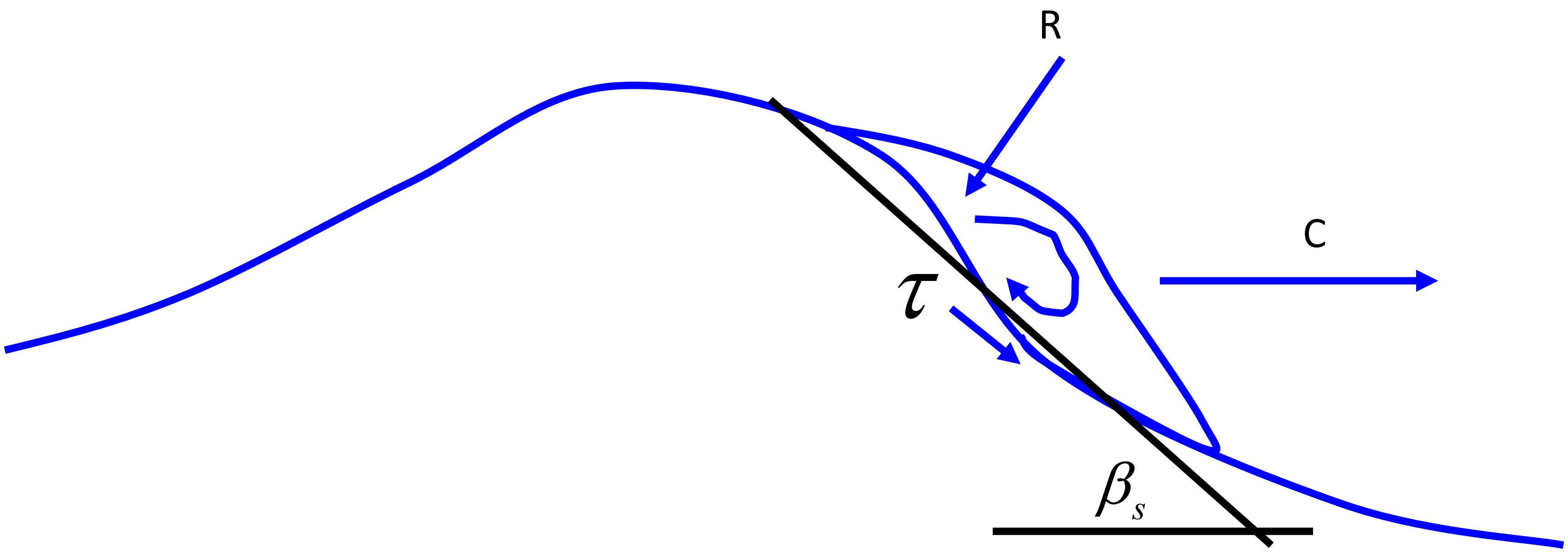
- Roller model
- Combined shear stress due to waves and current
- Wave-driven current

Roller model

- Delay between start of wave breaking and start of upward slope in setup
- Similar delay between start of wave breaking and wave-driven current
- This can be explained and modelled by a 'roller model'

Roller model

- Energy in roller denoted by E_r



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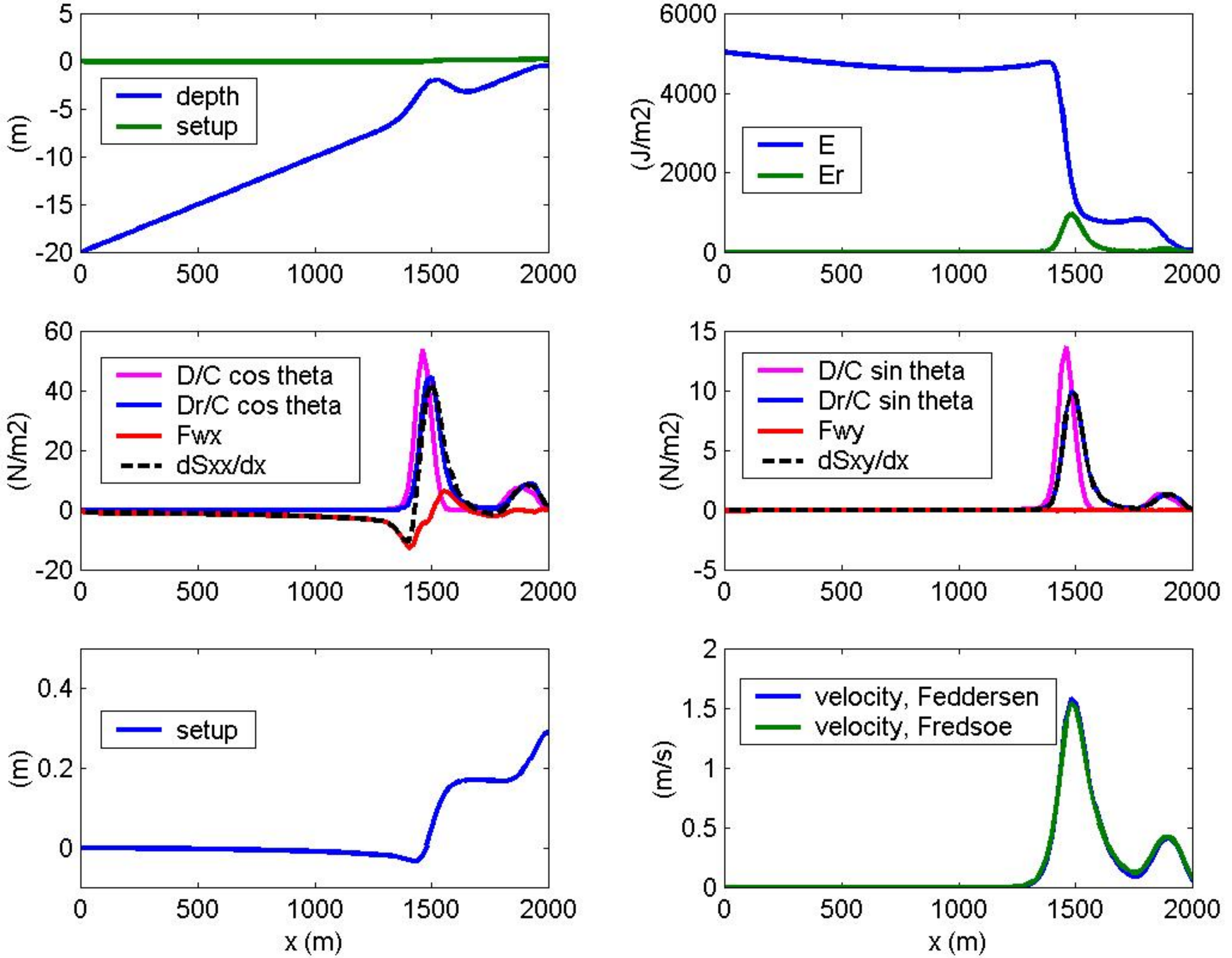
Roller balance

$$\frac{dE_r}{dt} = \frac{\partial E_r}{\partial t} + \frac{\partial E_r C \cos\vartheta}{\partial x} + \frac{\partial E_r C \sin\vartheta}{\partial y} = D_w - D_r$$

$$D_r = 2g\frac{\beta_s}{\beta_2}\frac{E_r}{C}$$

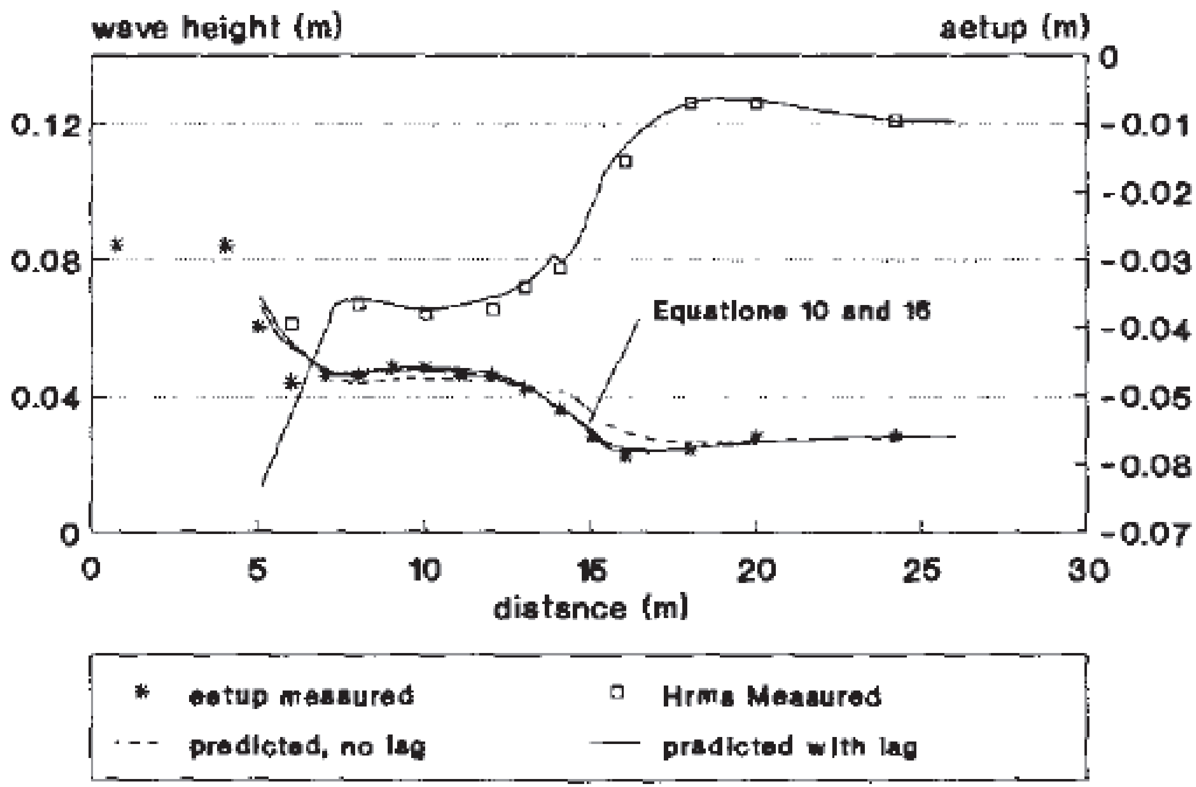
Dissipation of wave energy feeds into roller energy

Example of longshore and cross-shore forcing, flow and setup



Effect of roller on setup

Fig. 5 - Random Wave Height and Setup
Battjes and Janssen (1978) Test HJ12



Nairn, Roelvink and Southgate , ICCE 1990

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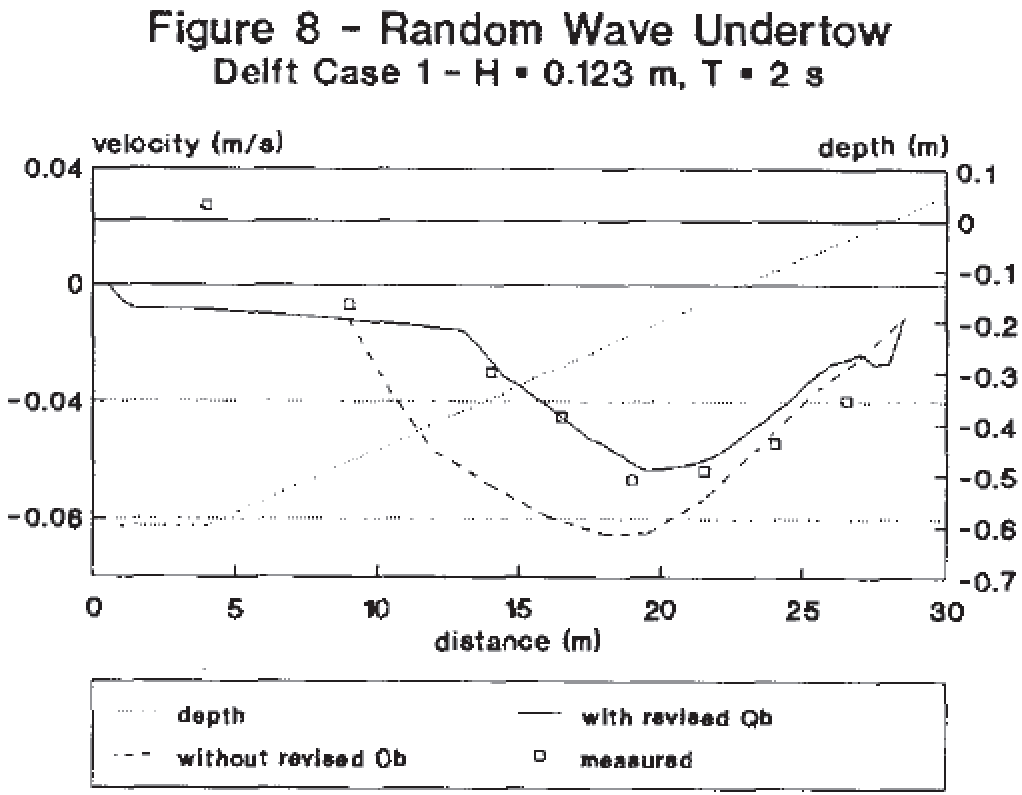
Effect of roller on undertow



Bed shear stress due to waves and currents

- Important for
  - Generation of longshore currents
  - Sediment transport (stirring)

- Subjects:
  - Bottom shear stress uniform flow
  - Shear stress waves only
  - Shear stress due to waves and currents

Bed shear stress

- Current only

$$\tau_c = \rho g \frac{v^2}{C^2} = \rho C_f v^2$$

Orbital velocity amplitude near bottom

- Waves only

$$\tau_w = \frac{1}{2} \rho f_w u_w^2, \qquad u_w \approx u_{rms} \sqrt{2}$$

Orbital excursion

$$f_w = 1.39 \left(\frac{A}{z_0} \right)^{-0.52}, \qquad A = \frac{u_w T_p}{2\pi}$$

Friction factor

Bed roughness=k_s/30

Near-bed velocity

- For regular waves:

$$\hat{u}_w = \frac{\pi H}{T} \frac{1}{\sinh(kh)}$$

- For random waves:

$$\hat{u}_w = u_{rms}\sqrt{2} \approx \frac{\pi H_{rms}}{T_p} \frac{1}{\sinh(k_p h)}$$

Example

$h = 3$ m
$r = 0.06$ m
$\bar{v} = 1$ m/s
$H = 1.18$ m
$T = 8$ s

τ_c ?

τ_w ?

$$\tau_c = \rho g \frac{v^2}{C^2} \leftarrow$$

$$C = 18 \log\left(\frac{12h}{r}\right) \rightarrow 50\ \mathrm{m}^{1/2}/\mathrm{s}\ \text{(Chézy)}$$

$$\tau_c = 3.9\ \mathrm{N/m^2}$$

$$\tau_w = \tfrac{1}{2} \rho f_w \hat{u}_o^2 \sin^2(\omega t)$$
$$\uparrow \quad \uparrow$$

$$\hat{u}_o = 1\ \mathrm{m/s} \rightarrow a_o = \frac{\hat{u}_o T}{2\pi} = 1.27\ \mathrm{m}$$

$$\frac{a_o}{r} = \frac{1.27}{0.06} = 21.2 \qquad f_w = f\left(\frac{a_o}{r}\right) = 0.048$$

$$\hat{\tau}_w = \tfrac{1}{2} \rho f_w \hat{u}_o^2 \rightarrow 22.5\ \mathrm{N/m^2}$$

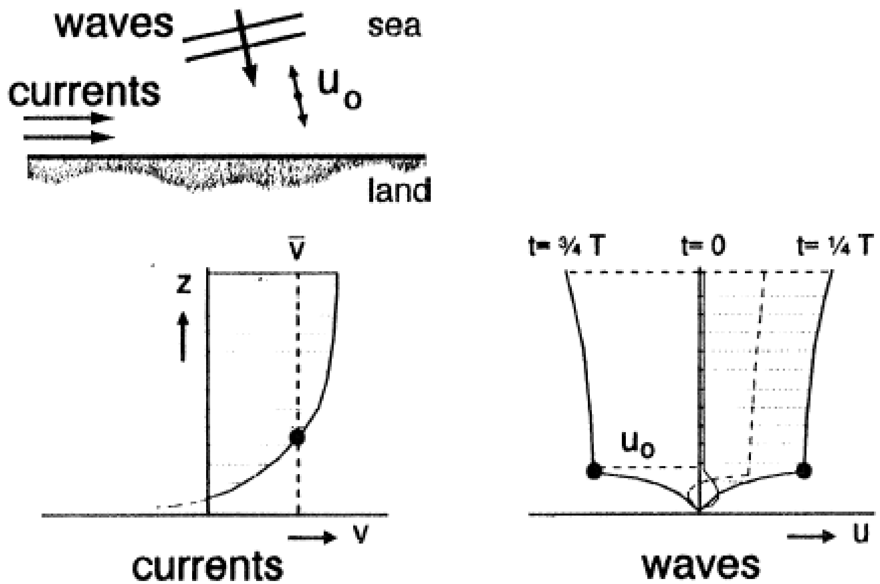
$$\bar{v} = 1\ \mathrm{m/s} \rightarrow \tau_c = 3.9\ \mathrm{N/m^2}$$
$$\hat{u}_o = 1\ \mathrm{m/s} \rightarrow \hat{\tau}_w = 22.5\ \mathrm{N/m^2}$$

$$u_0 = \hat{u}_w = \frac{\pi H}{T} \frac{1}{\sinh(kh)}$$

$$f_w = 1.39 \left(\frac{A}{z_0}\right)^{-0.52}$$

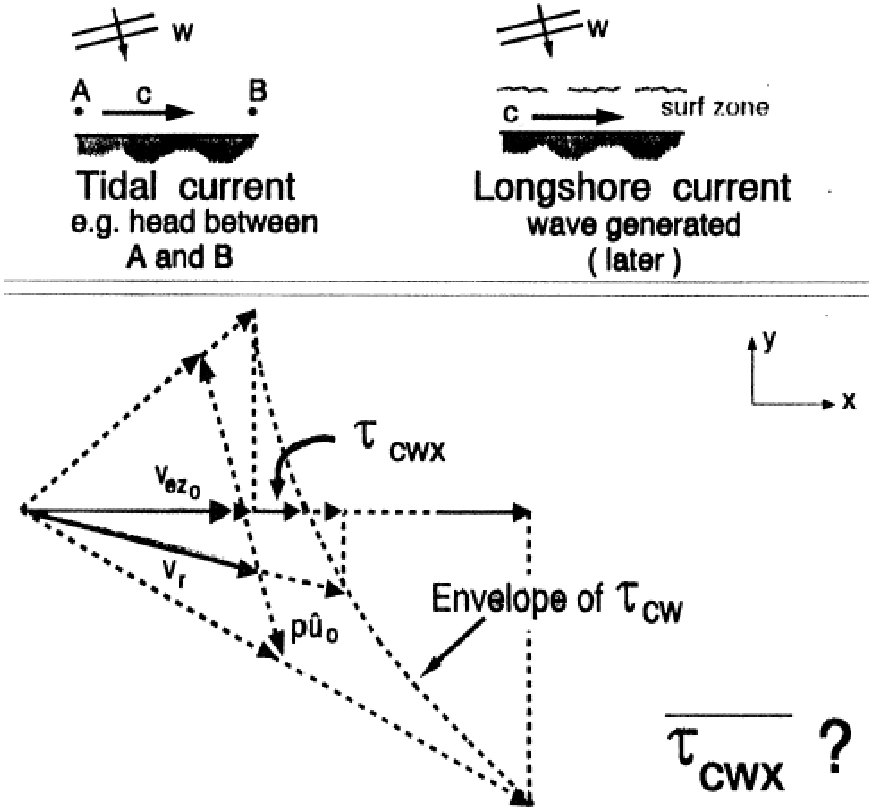
Wave-current interaction

- Given current shear stress and wave shear stress, what is combined wave-current shear stress?



Mean shear stress

- Shear stresses cannot be just added up;
- Neither can near-bed velocities



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Bed shear stress due to currents and waves

- Various approaches since ’60s (Bijker)
- Soulsby (e.g. ‘Dynamics of Marine Sands’) provides parameterized version of simple and more advanced models
- Maximum shear stress in wave period (important for stirring up sediment) $\tau_{\max}$
- Mean shear stress in wave period (important for flow resistance), τ_m also often called τ_{cw}

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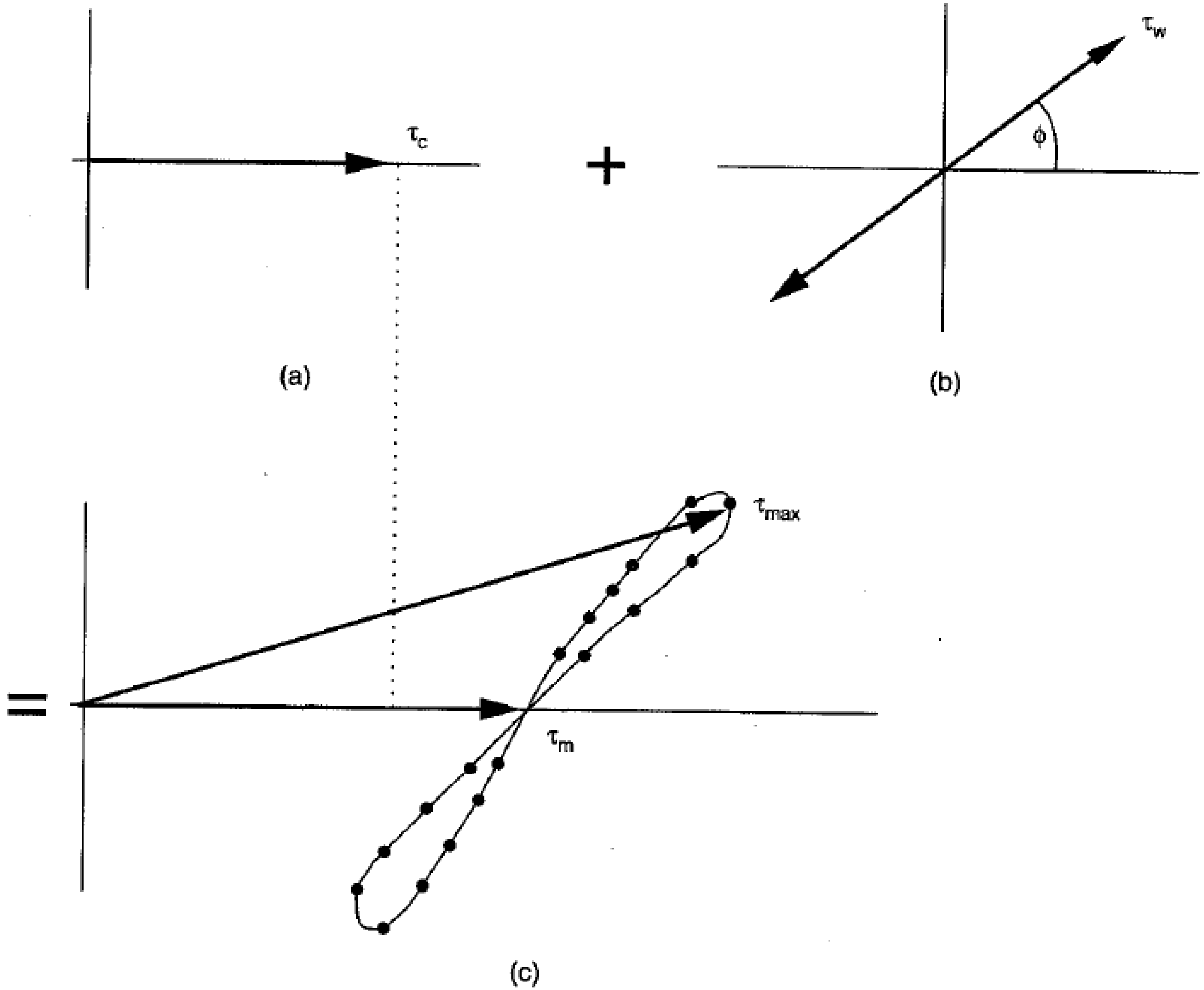
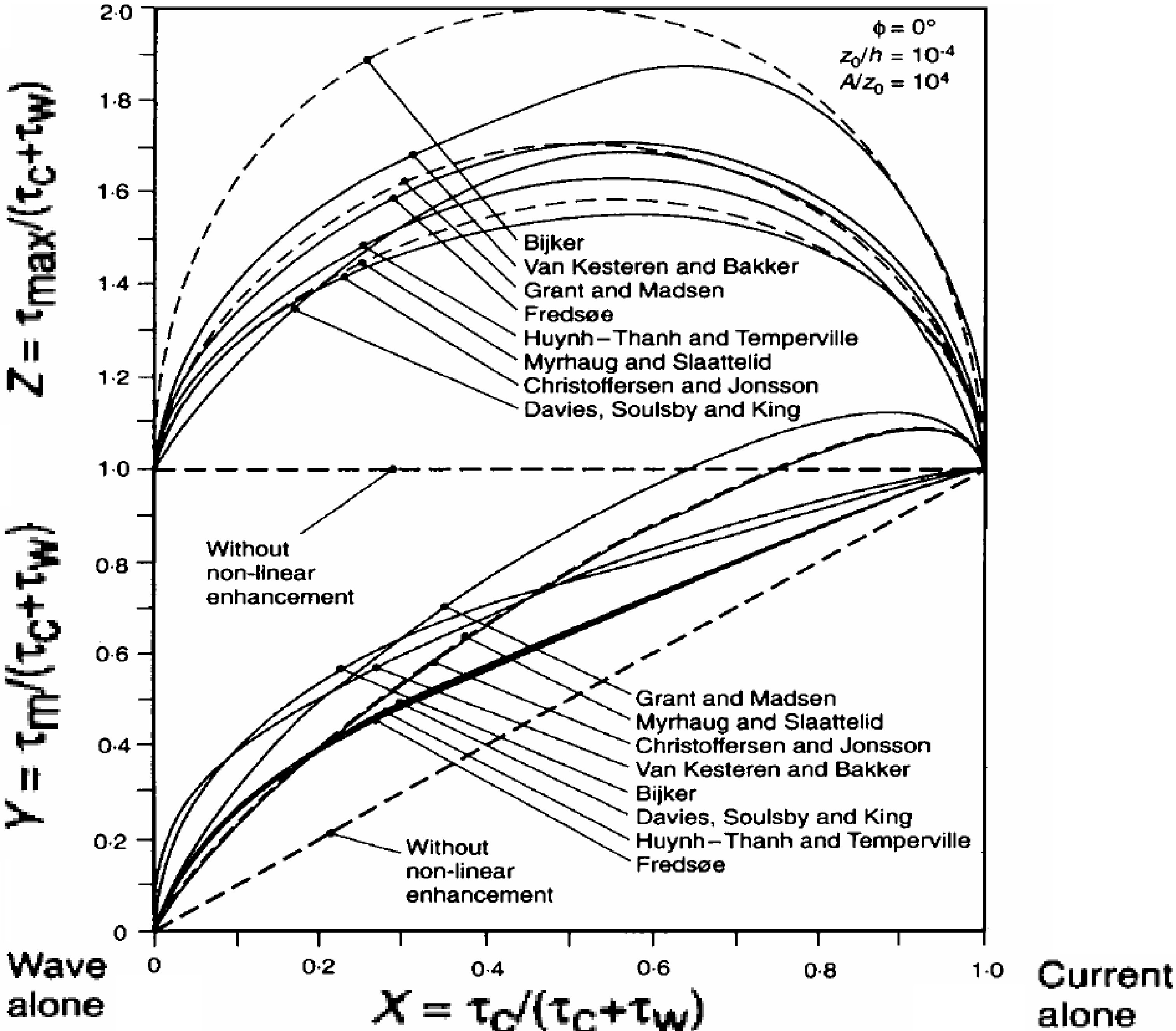


τ_c
+
τ_w
ϕ
(a)
(b)
=
τ_{max}
τ_m
(c)



τ_C = bed shear stress due to current alone
τ_W = amplitude of bed shear stress due to waves alone

IH DELFT

Simple expressions

- Soulsby DATA2:

$$\tau_m = \tau_c \left[1 + 1.2 \left(\frac{\tau_w}{\tau_c + \tau_w} \right)^{3.2} \right]$$

$$\tau_{\max} = \sqrt{\left(\tau_m + \tau_w \cos\varphi\right)^2 + \left(\tau_w \sin\varphi\right)^2}$$

```
% function to compute tauc for given taum
function [tauc]=soulsby(taum,tauw)
tauc=0;taucold=1000;iter=0
while abs(taucold-tauc)>1e-6
   taucold=tauc
   tauc=taum/(1+1.2*(tauw/(tauc+tauw))^3.2)
   iter=iter+1
end
```

Bed shear stress according to Feddersen et al (2000)

Velocity moment

$$\tau_{cw} = \rho c_f \left\langle \left|\vec{u}\right| v \right\rangle$$

$$\langle |\vec{u}| v \rangle = \sigma_T \overline{v} \left[1.16^2 + (\overline{v}/\sigma_T)^2\right]^{1/2}$$

$$\sigma_T = u_{rms} = \frac{\hat{u}_w}{\sqrt{2}}$$

Assignment (2)

- Local Wave conditions:
  - Hrms = 1 m
  - Tp = 6 s
  - Direction 20 deg to shore normal
  - Depth 2 m
  - Roughness r=0.06
- Compute
  - wave dissipation rate,
  - longshore wave force Fy,
  - mean longshore shear stress,
  - wave-induced shear stress tauw,
  - current-induced shear stress tauc,
  - current velocity v

Longshore current computation

- Snel's Law
- Wave energy balance and roller energy balance
- Bed shear stress due to currents and waves
- Good example of field validation: Ruessink et al., 2001.
- Egmond (NL) and Duck (USA)

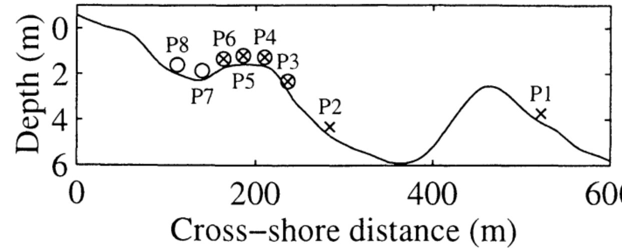


Figure 2. Depth relative to mean sea level versus cross-shore distance on October 16, 1998, at Egmond and locations of current meters (circles) and pressure sensors (crosses).

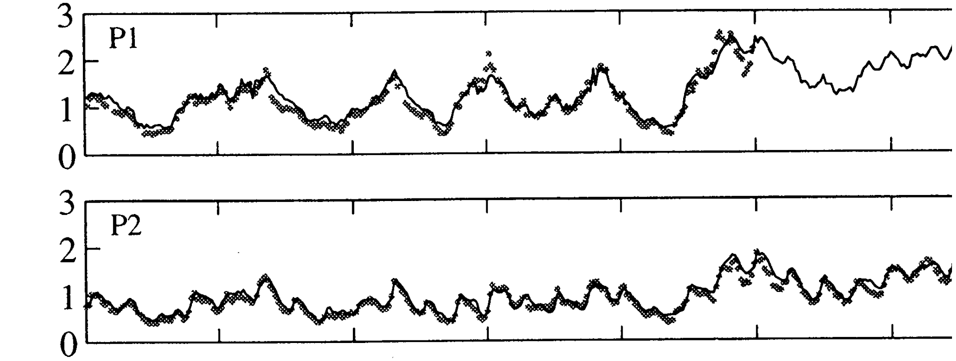
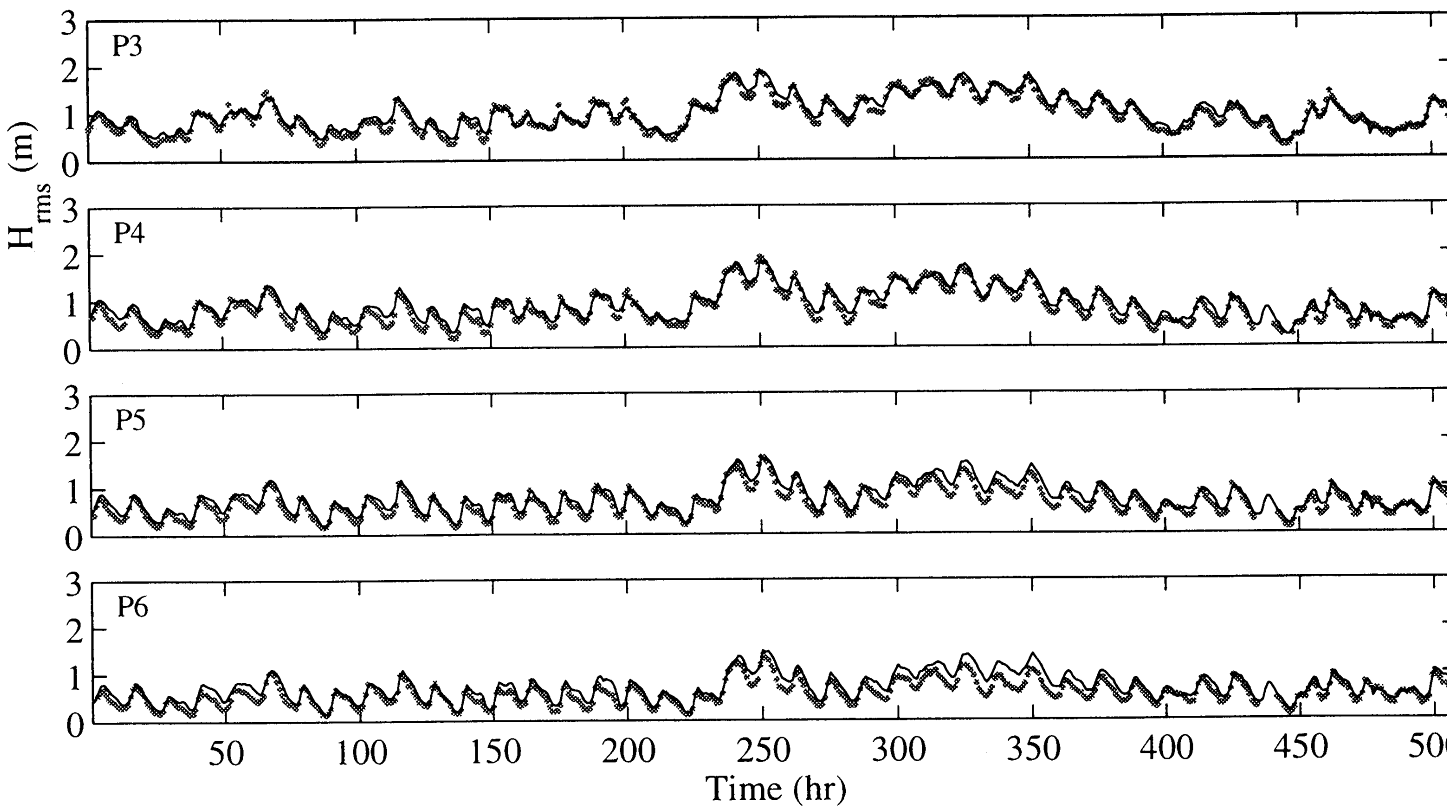



Figure 5. Measured (symbols) and modeled (lines) H_{rms} from offshore (P1) to onshore (P6) versus time at Egmond.

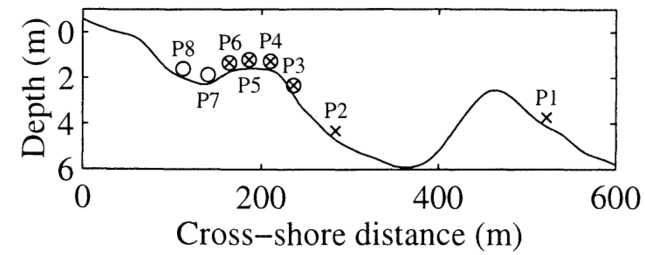


Figure 2. Depth relative to mean sea level versus cross-shore distance on October 16, 1998, at Egmond and locations of current meters (circles) and pressure sensors (crosses).

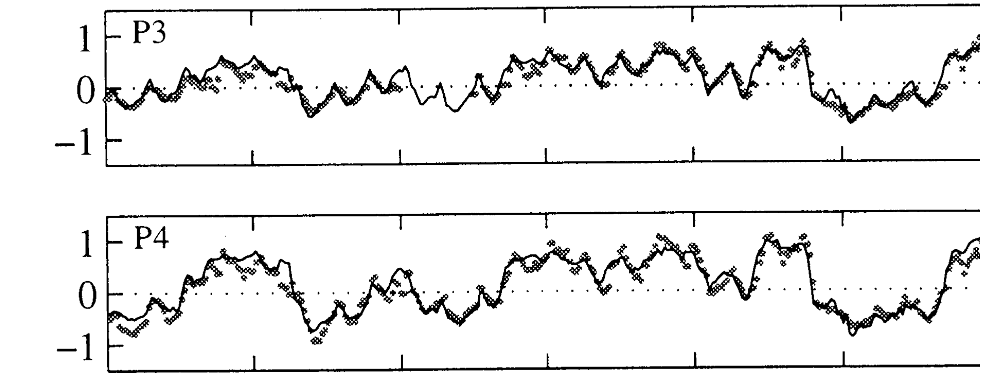
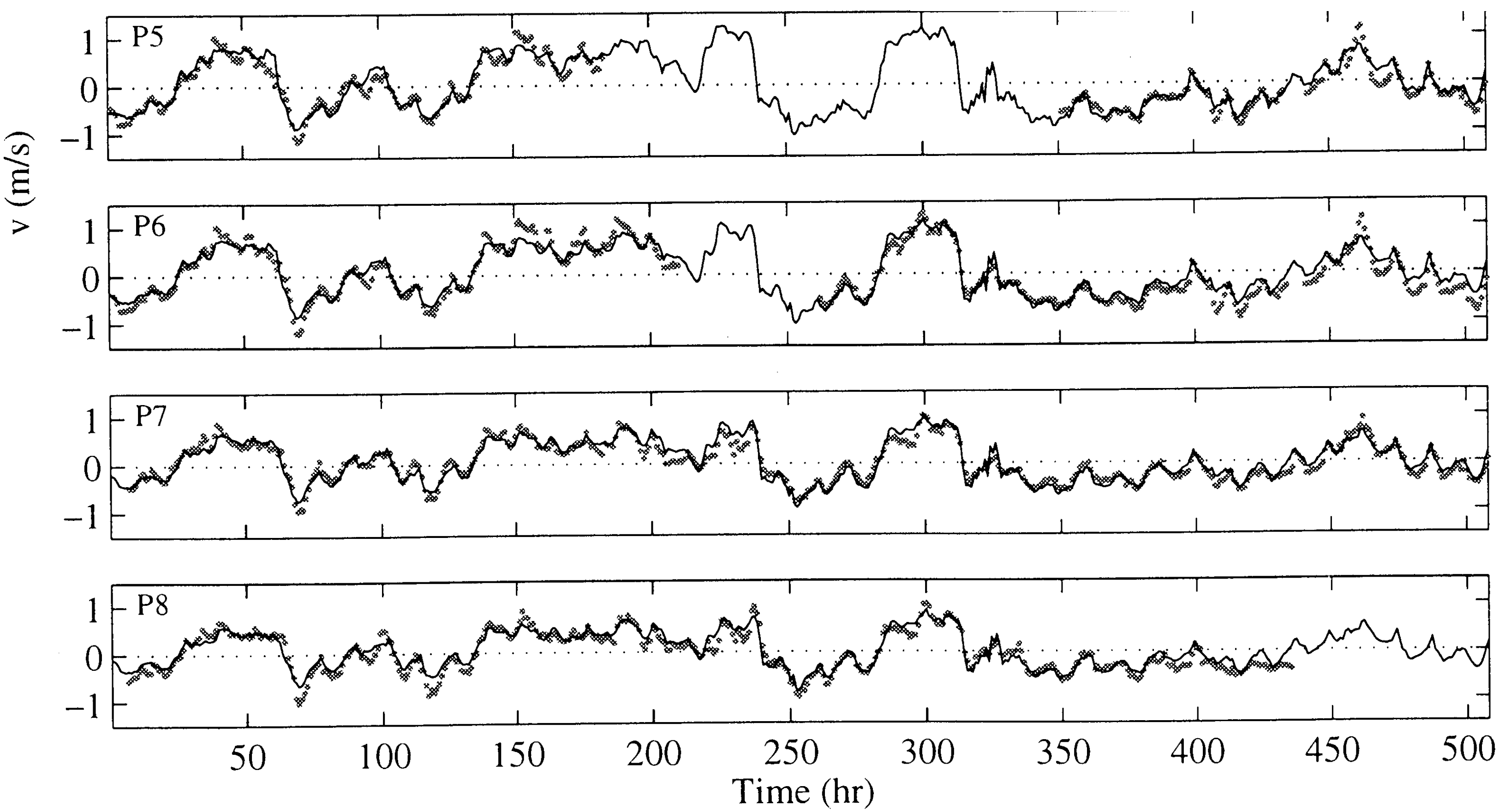



Figure 7. Measured (symbols) and modeled (lines) $\overline{v}$ from offshore (P3) to onshore (P8) versus time at Egmond. Error statistics are given in Table 1, roller run.

Questions to think about

- Study Ruessink et al (2001) paper
- Do you recognize the formulations?
- What are the main differences with the formulations discussed in Roelvink&Reniers?
- Which model parameters are most important for model skill?