Free Surface Hydrodynamics
2DH and 3D Shallow Water Equations
Applications

Prof. Dano Roelvink

UNESCO IHE DELFT

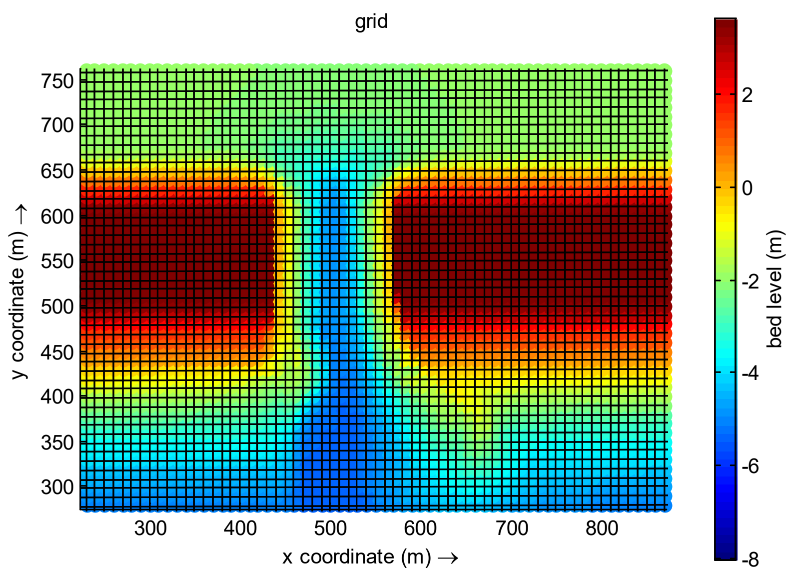
Deltares
Enabling Delta Life

Numerical models

- Grid types
  - Rectilinear, curvilinear, unstructured
- Discretization
  - Finite difference, finite volume, finite elements
- Solution methods
  - Implicit vs explicit
  - Explicit: hard stability criterion $c\frac{\Delta t}{\Delta x} < 1$

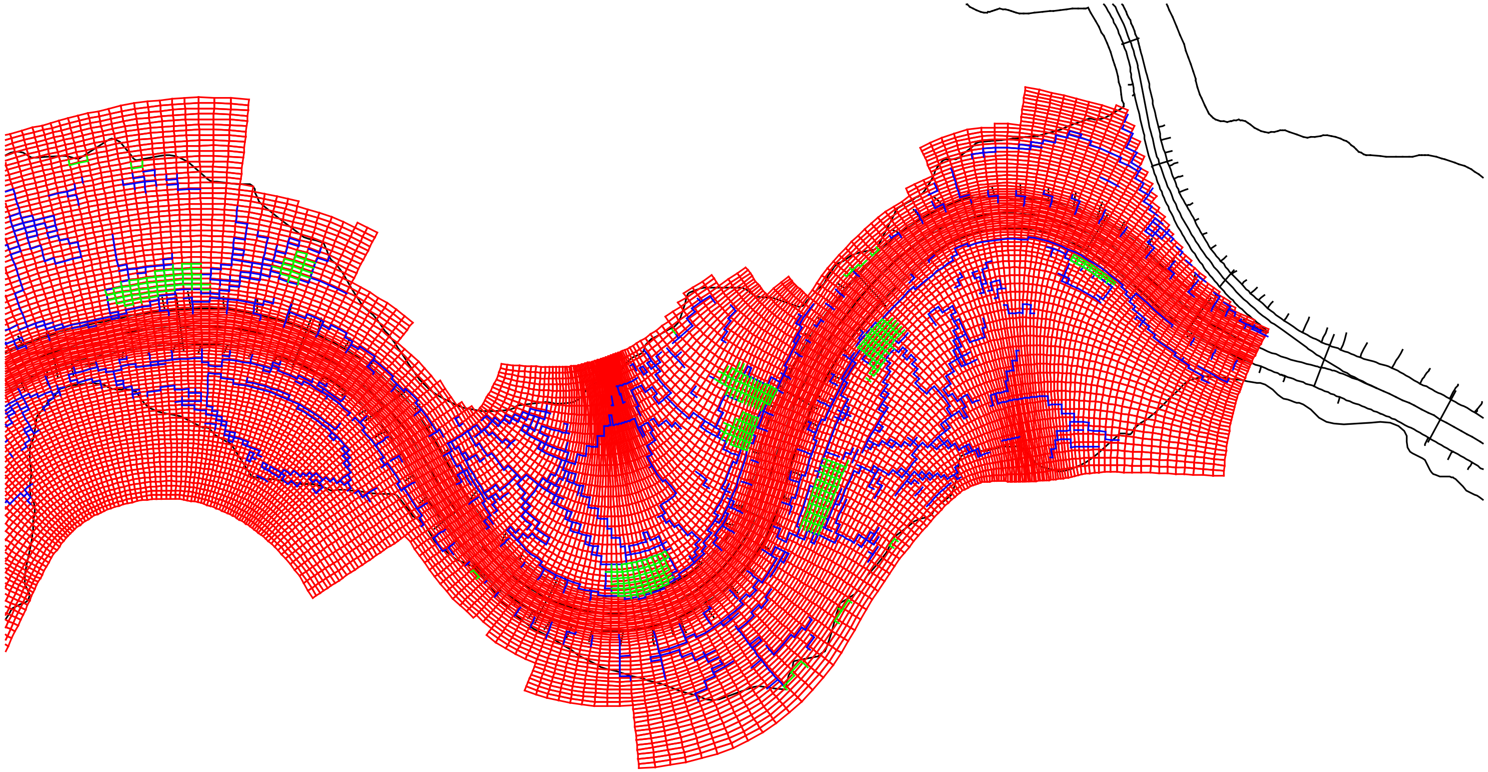
Grid types

- Rectangular



Grid types

- Curvilinear, structured



Grid types

- Unstructured, 'Flexible Mesh'

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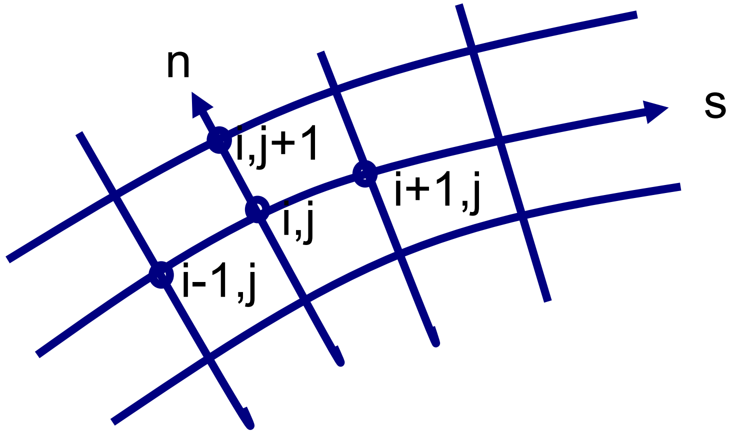
Numerical methods

- Finite difference
- Finite volume
- Finite element

Finite difference

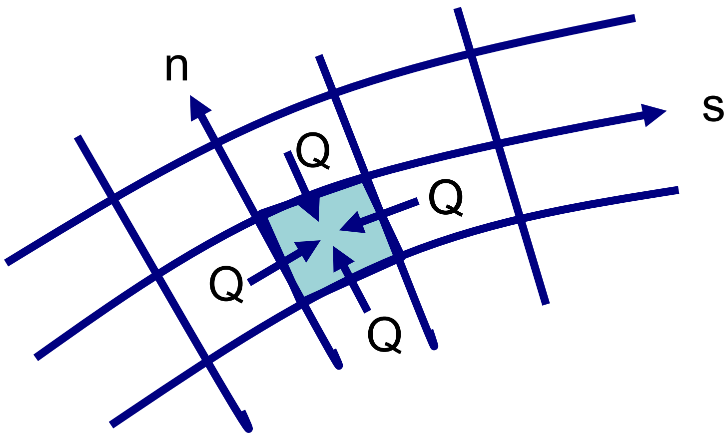
- From partial derivatives to partial differences
- For example: the gradient of c in s-direction
- Easy to turn a PDE into a finite difference scheme
- Only works on structured grids



$$\left.\frac{\partial c}{\partial s}\right|_{i,j} = \frac{c_{i+1,j} - c_{i-1,j}}{s_{i+1,j} - s_{i-1,j}}$$

Finite volume

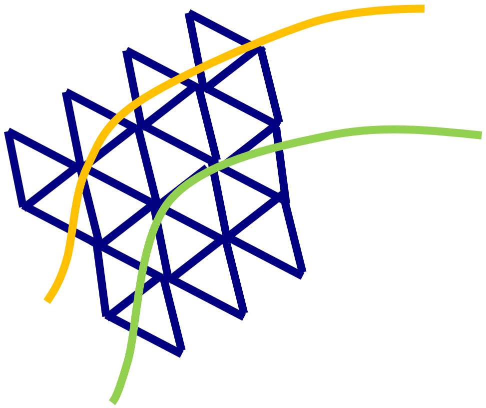
- Change of contents of a volume is sum of fluxes into volume
- Great for conservation laws
- Example: change of volume of a grid cell
- More difficult derivation
- Can be applied on structured and unstructured grids



$$\frac{\Delta V}{\Delta t} = \sum Q_{in}$$

Finite element

- Divide model up in small elements, often triangles
- Take field equations, PDEs
- Approximate the field within each element by a function
- Assemble contribution from all elements into a matrix
- Solve matrix
- Mathematically quite complex

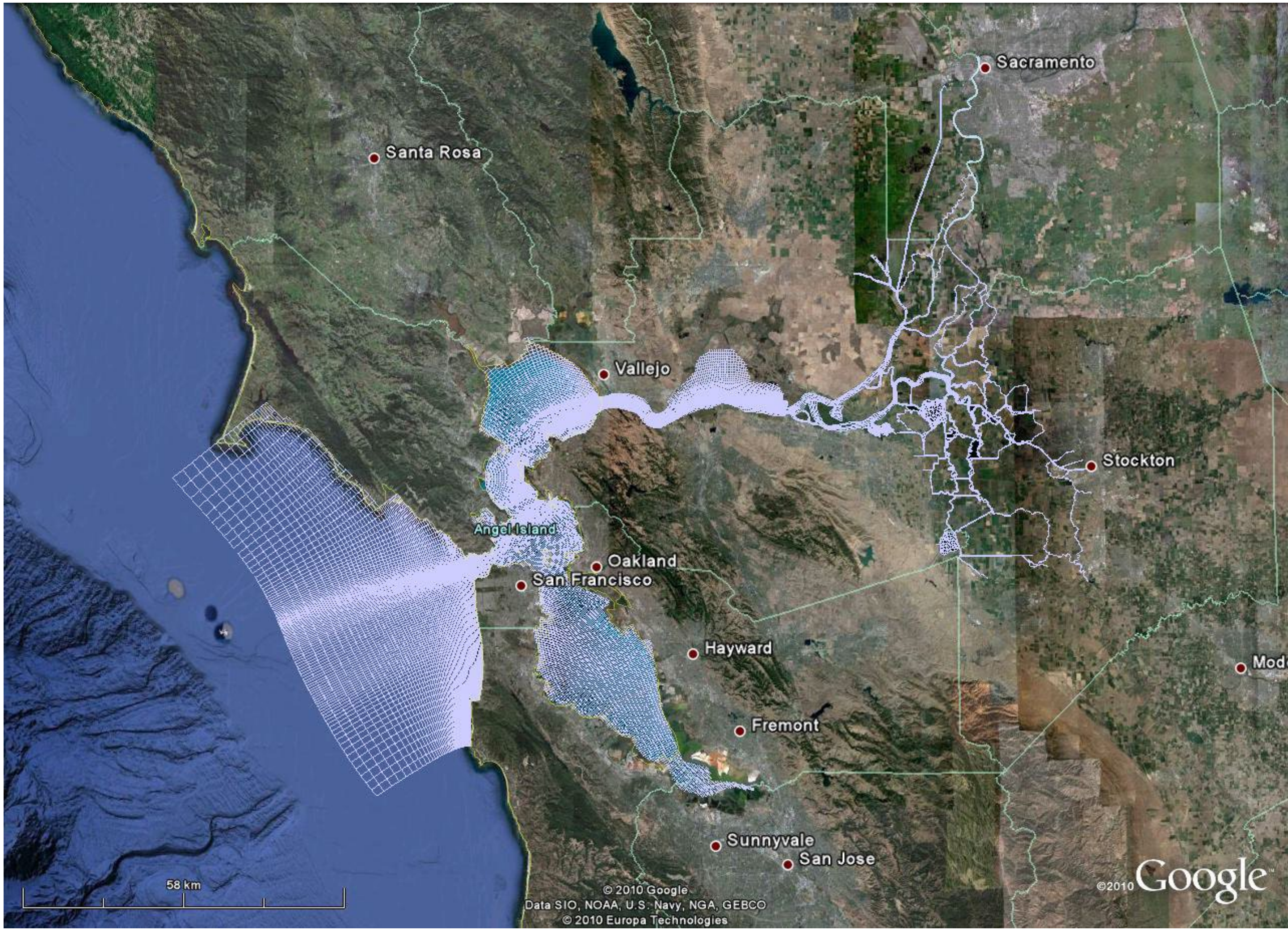


Applications

- San Francisco Bay & Delta model
- Mekong delta flows, tides, salinity, sediment
- Tidal current modelling (Texel, Singapore)
- Storm surge prediction (Hurricane Ike, North Sea)
- Detailed river modelling (Rhine branches)
- Flooding (USA)
- Water quality modelling
- Morphology modelling (Bangladesh)

Delta Dflow-FM Hydrodynamic Model

- Developed by IHE and Deltares for SF Bay Delta
- New hybrid grid
- 3-dimensional, ocean-to-river
- <u>Houses:</u>
  hydrodynamics
  salinity
  temperature
  sediment
  phytoplankton
  bivalves



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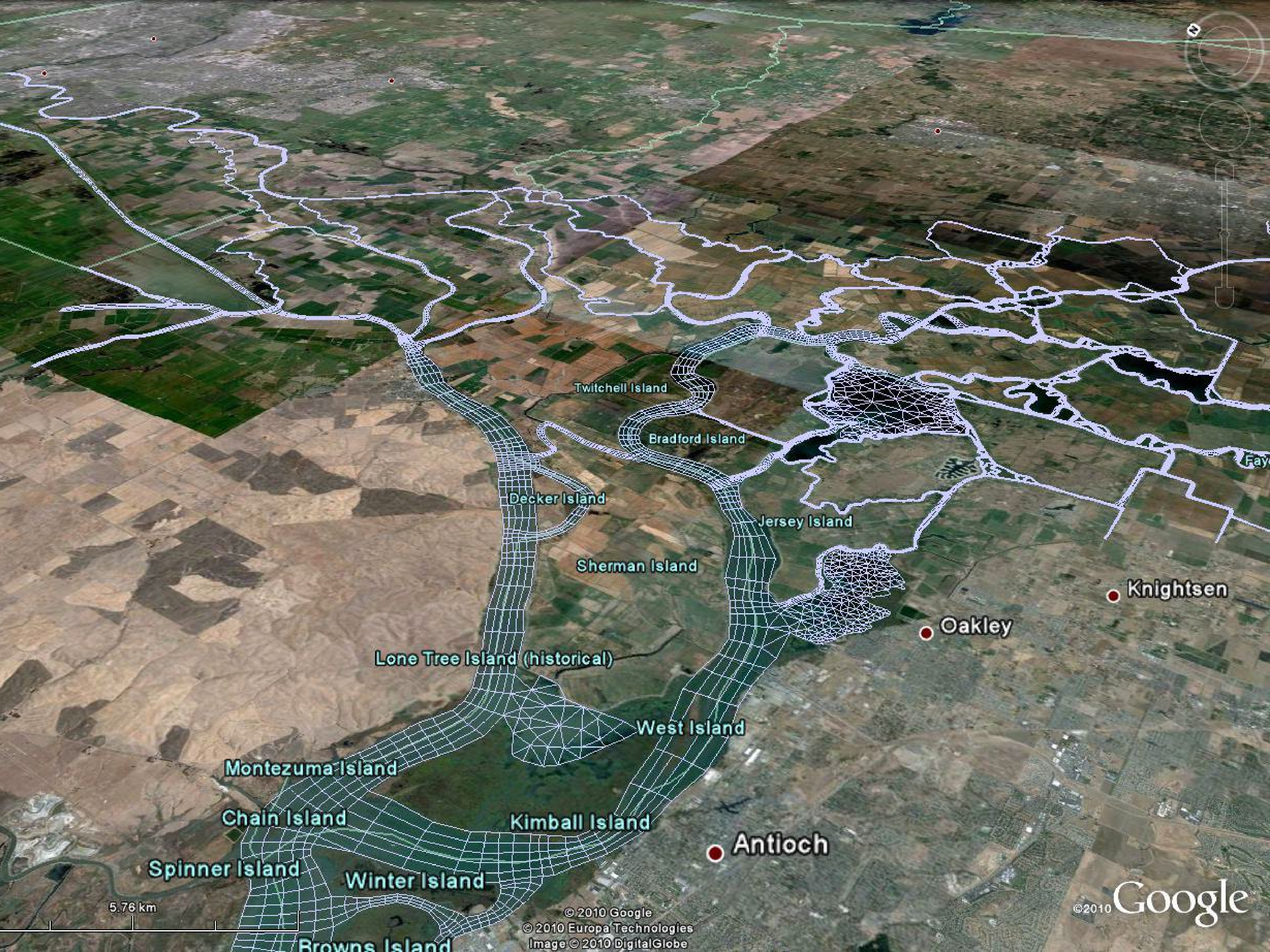

Twitchell Island
Bradford Island
Decker Island
Jersey Island
Sherman Island
Knightsen
Oakley
Lone Tree Island (historical)
West Island
Montezuma Island
Chain Island
Kimball Island
Antioch
Spinner Island
Winter Island
Browns Island
5.76 km
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Image © 2010 DigitalGlobe
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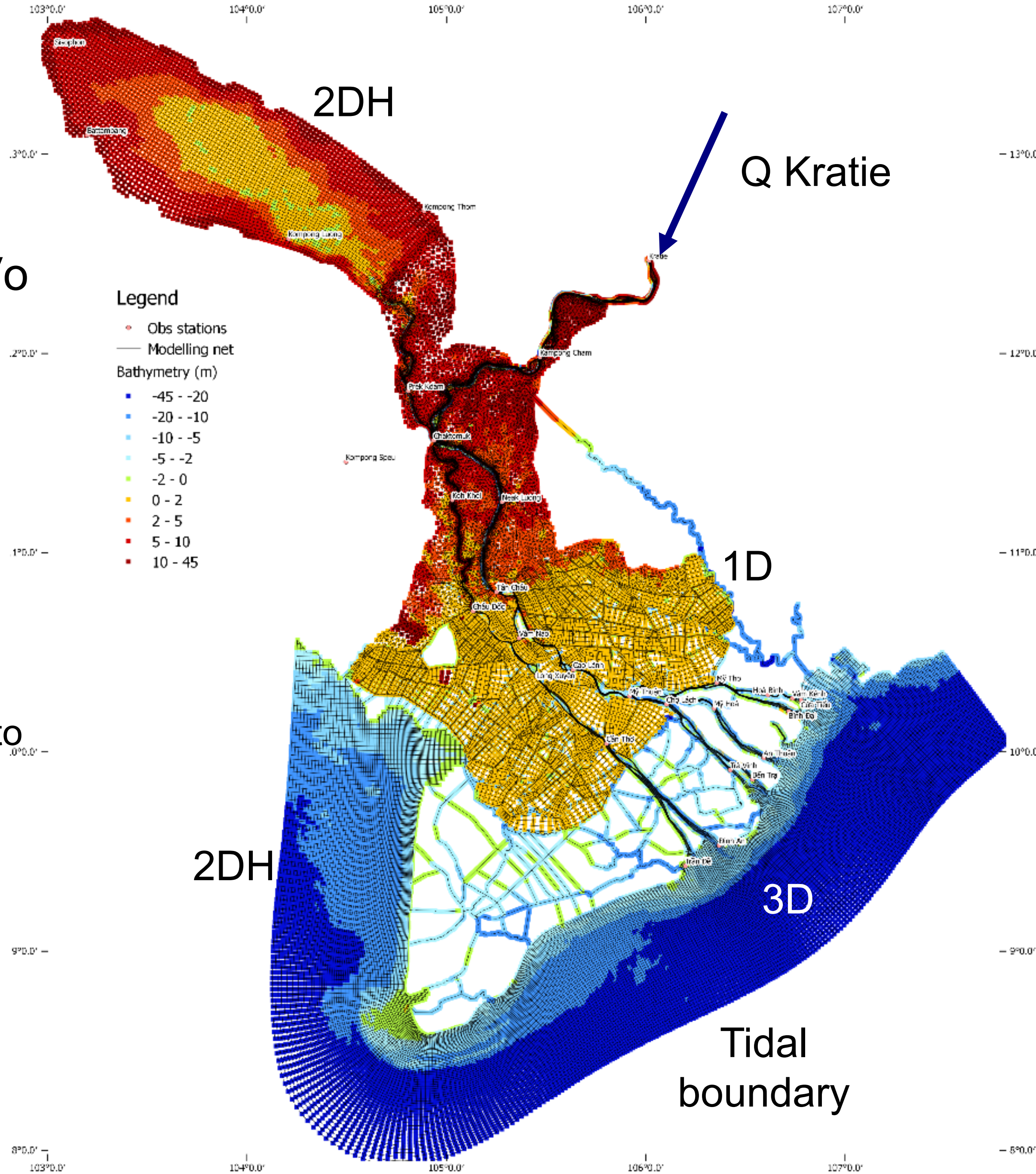
Tidal propagation

Mekong delta

- PhD work Thanh Vo
- Part of large ONR project on tropical deltas
- Issues:
  - Flow distribution
  - Water levels
  - Seasonal variation
  - Sediment delivery to coast
  - Salinity intrusion
- Approach:
  - 1D-2DH coupling
  - 3D for coast and estuaries



Bassac river outflow

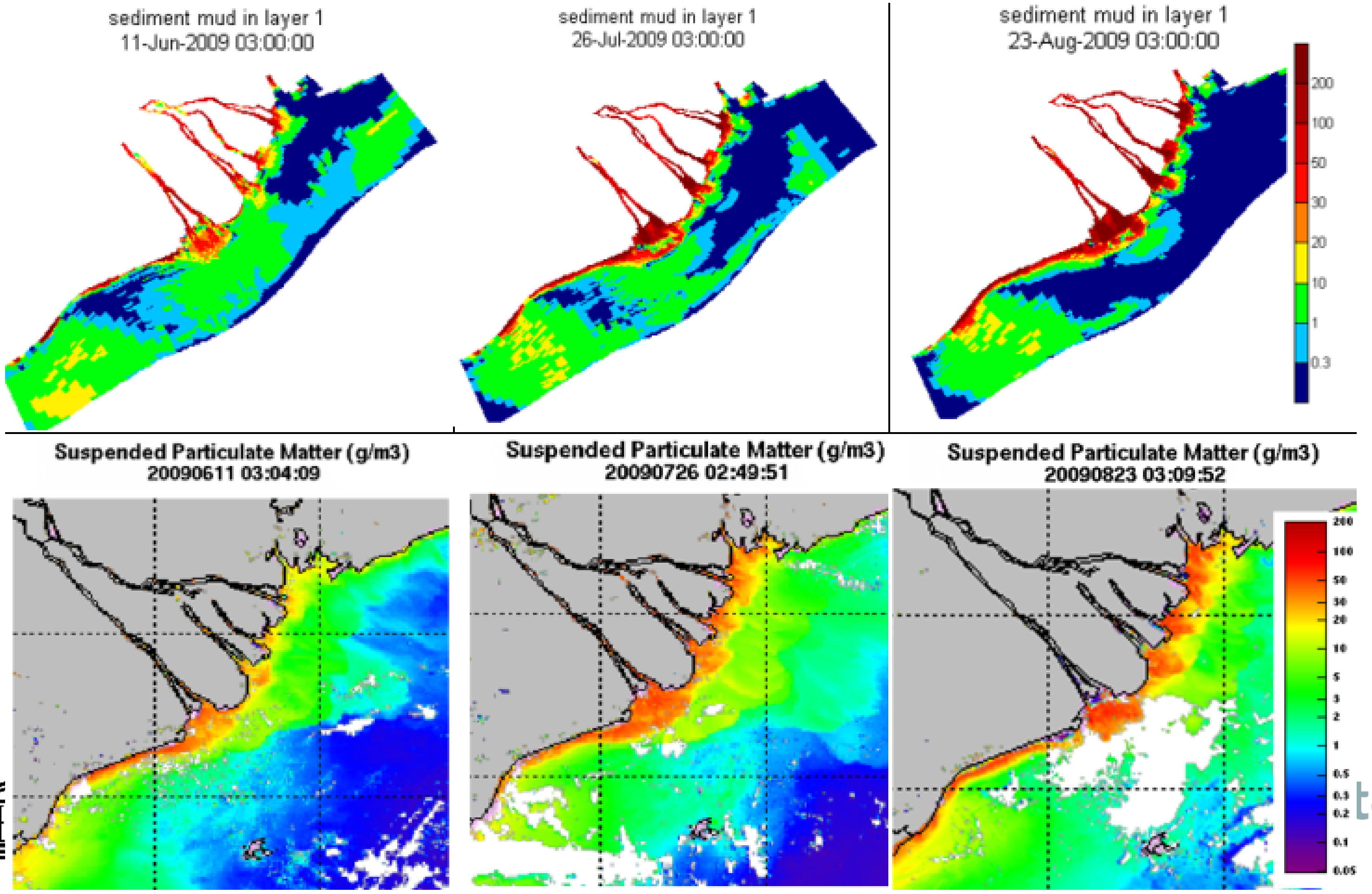
- 3D flow and salinit
- Fresh water plume
- Saline intrusion
- Interaction tide and discharge

3D Model of shelf and estuaries

- Structured 3D Delft3D model covers 7 estuarine branches in the Mekong Delta
- Forced by tides, river flow, wind and waves
- Includes salinity and sediment transport processes.
- Validation by satellite imagery
- Derivation of upstream boundary conditions from unstructured delta model

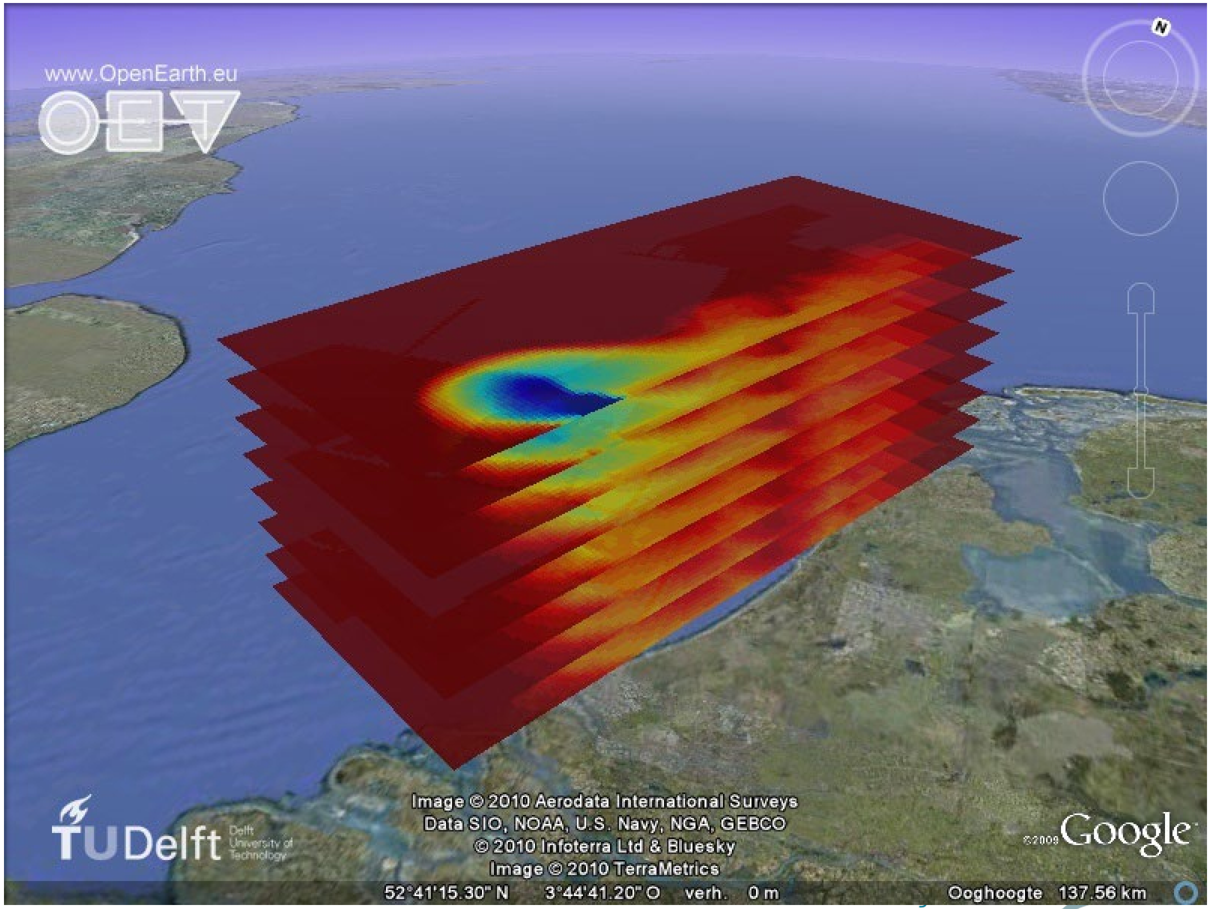
Sediment plumes

- Comparison of SSC with satellite imagery



Estuarine circulation

- See animations on www.openearth.nl

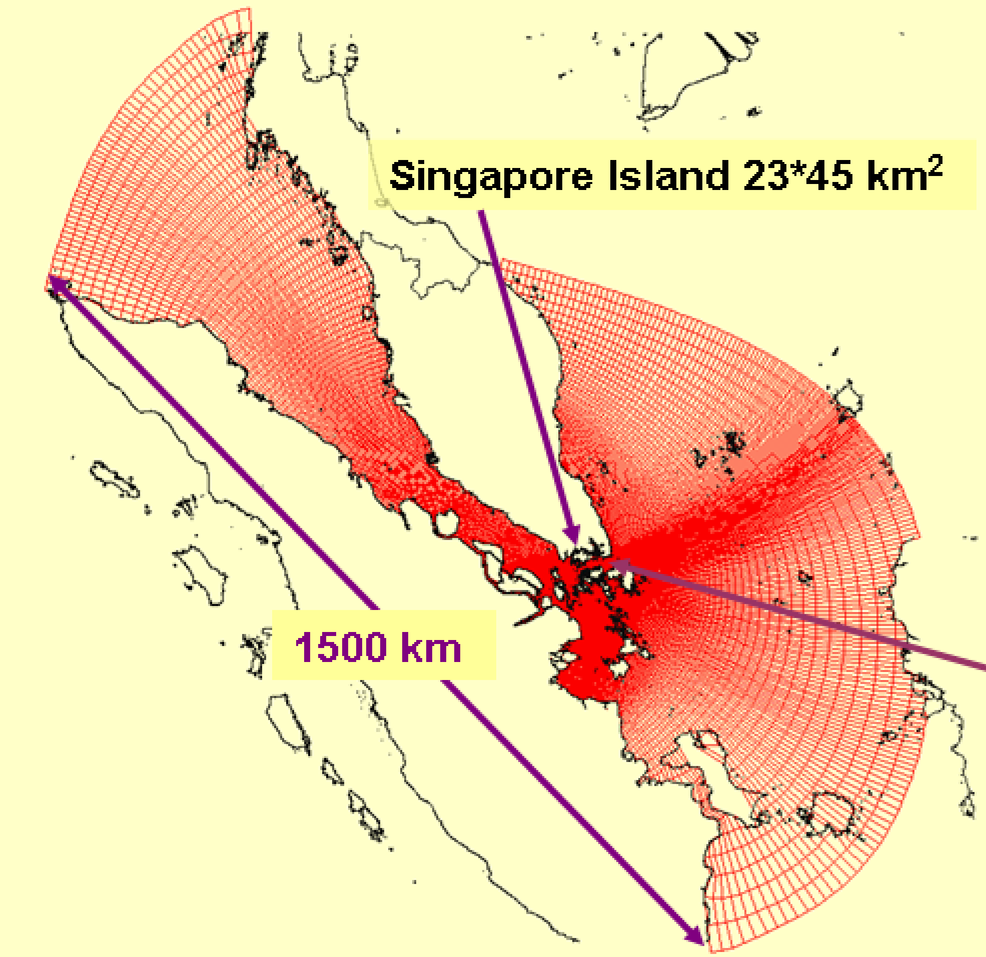


Texel, NL

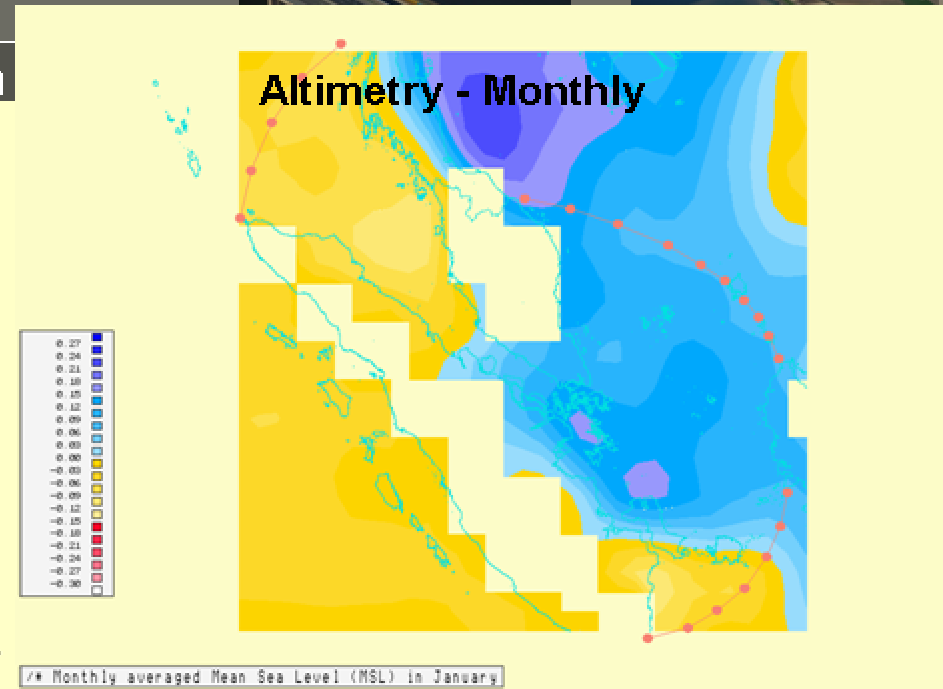
New Feature: Spatial Open-Sea Conditions by Satellite Altimetry

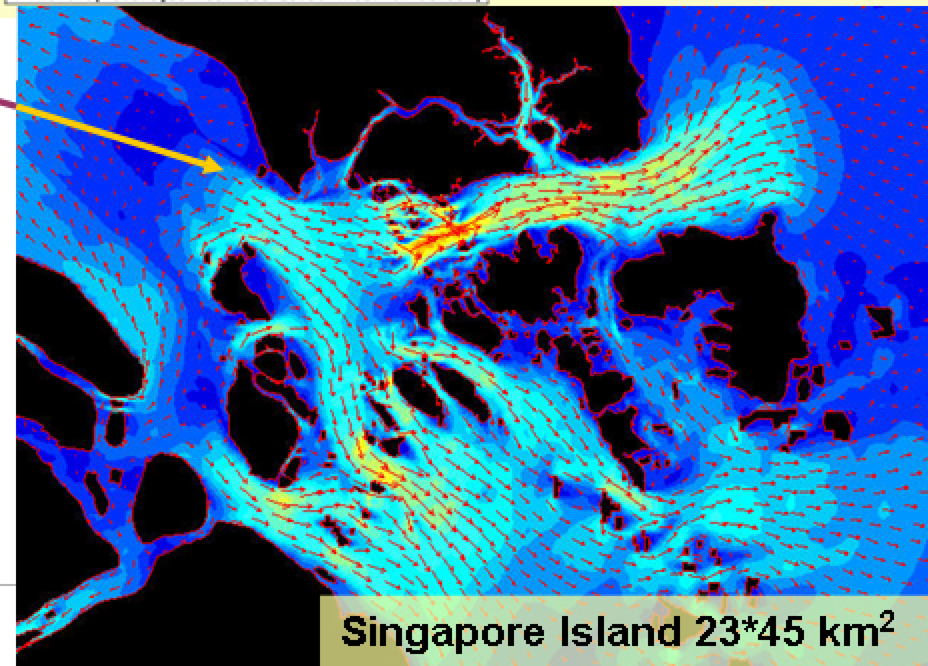
Singapore – SDWA – MustHave Box

Handling Ship Traffic – Land Reclamation



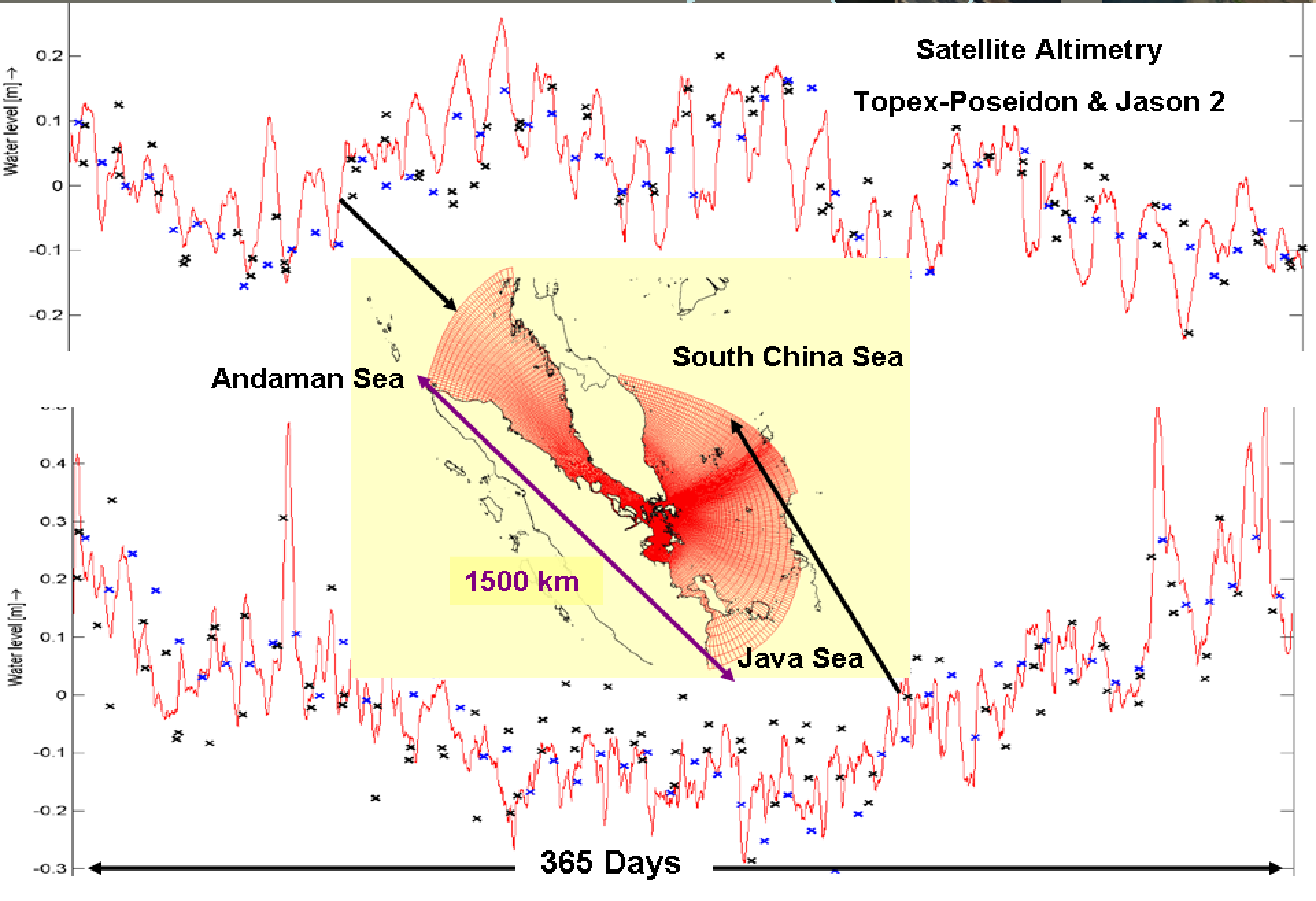
Malacca & Singapore Strait





New Feature: Add Sea Level Anomaly to Tide Levels

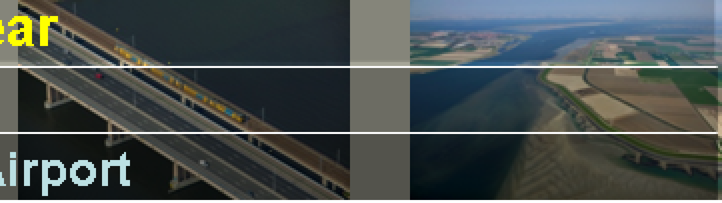
Research : Validation Satellite Altimetry – Tidal (Ground) Stations

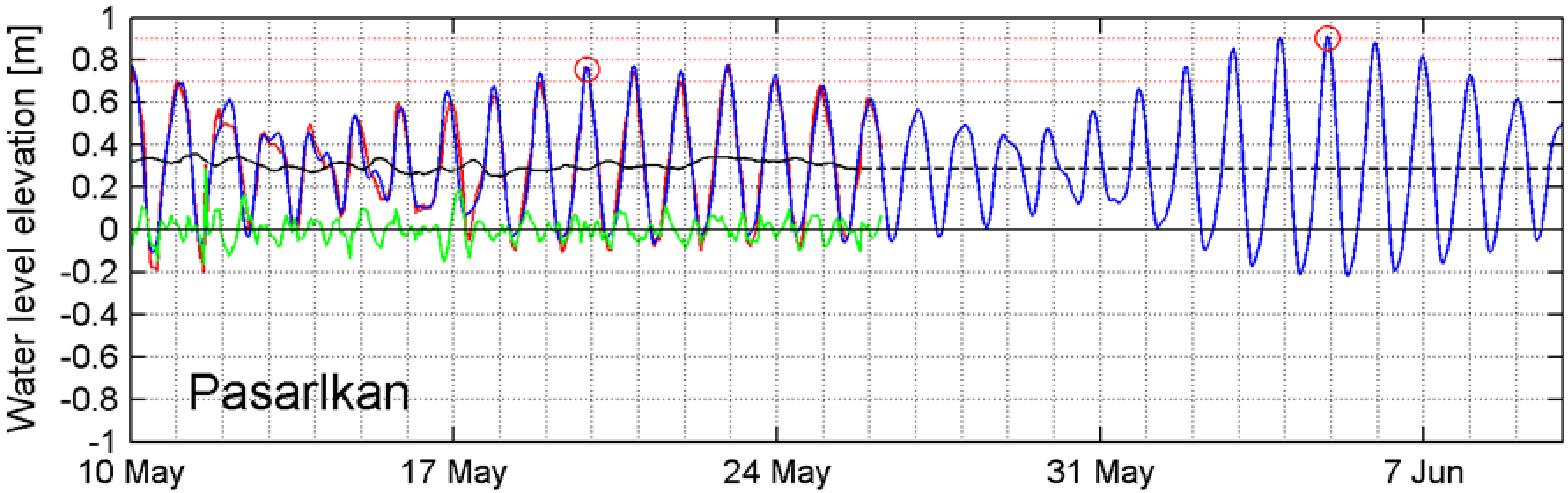


Projects : Land Subsidence Jakarta > 3cm / Year

Clients : Indonesia – World Bank

Research : Prediction Flooding Jakarta' Roads to Airport

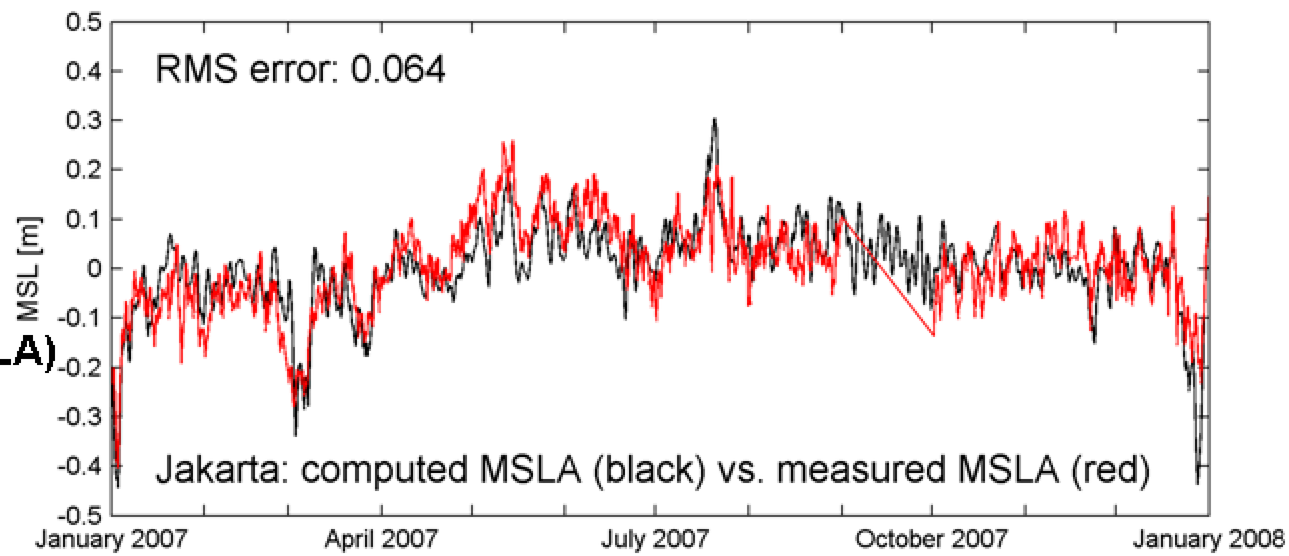


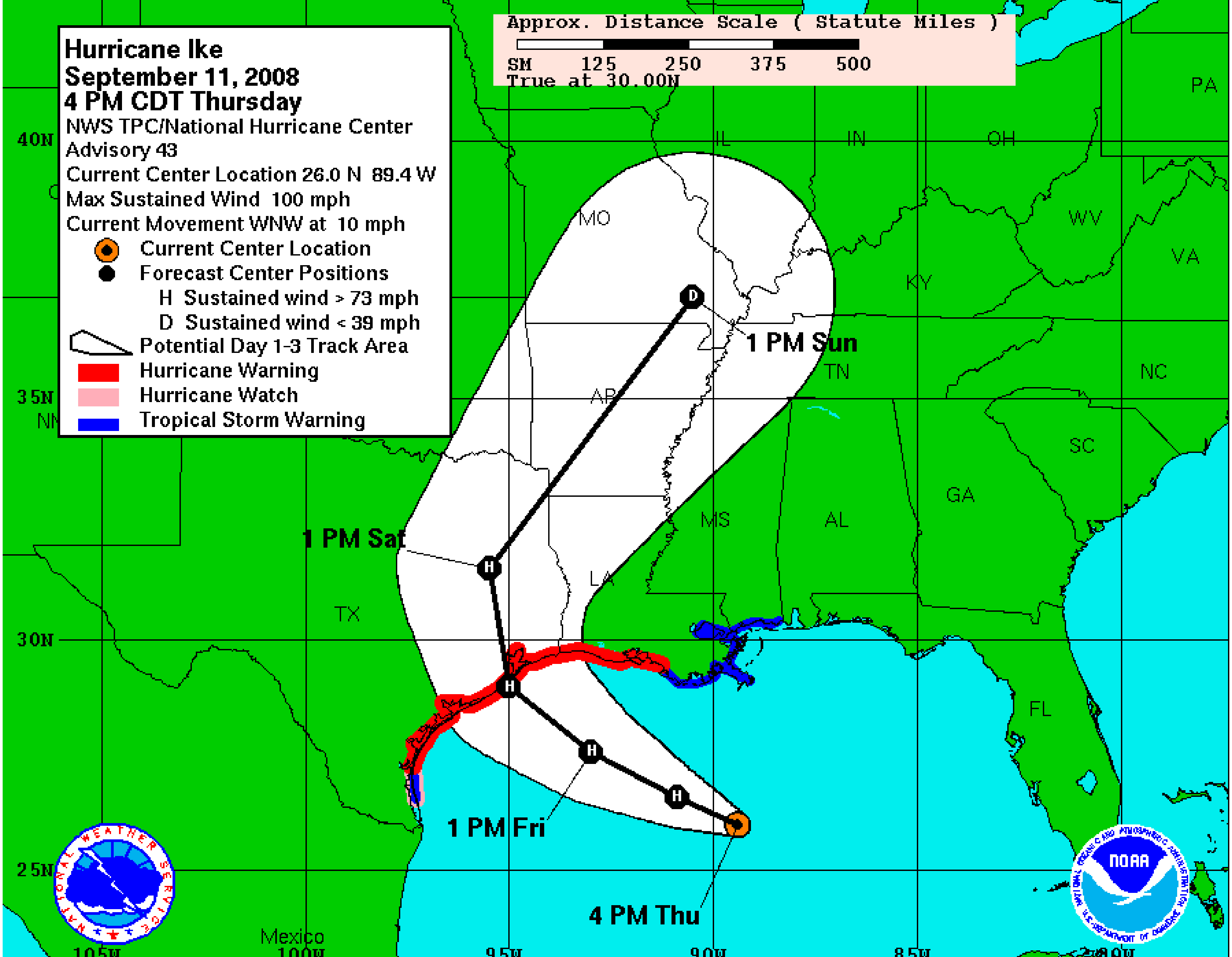


Mean Sea Level Anomaly (MSLA)

Due to Wind and Air Pressure

Over South China Sea Model




Hurricane Ike
September 11, 2008
4 PM CDT Thursday
NWS TPC/National Hurricane Center
Advisory 43
Current Center Location 26.0 N 89.4 W
Max Sustained Wind 100 mph
Current Movement WNW at 10 mph
Current Center Location
Forecast Center Positions
H Sustained wind > 73 mph
D Sustained wind < 39 mph
Potential Day 1-3 Track Area
Hurricane Warning
Hurricane Watch
Tropical Storm Warning
Approx. Distance Scale (Statute Miles)
SM 125 250 375 500
True at 30.00N
1 PM Sun
1 PM Sat
1 PM Fri
4 PM Thu
MO
IL
IN
OH
PA
WV
VA
KY
TN
NC
AR
SC
GA
MS
AL
LA
TX
FL
Mexico
40N
35N
30N
25N
105W
100W
95W
90W
85W
80W

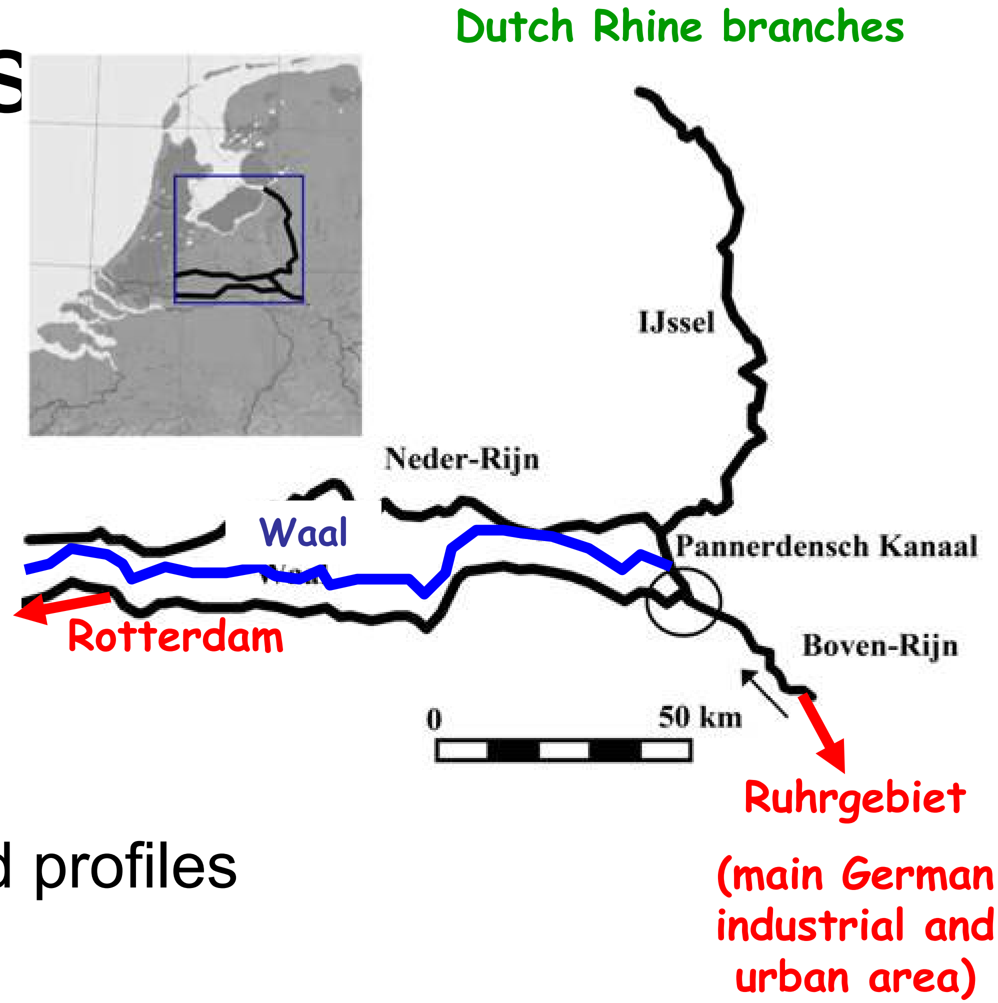
Example: Hurricane Ike

- A hydrodynamic model has been set up with the Delft3D system running in 2D mode. The hurricane track used in this model was downloaded from http://weather.unisys.com/hurricane/ .
- The model predicts surge levels of more than 5 metres above mean sea level in both San Antonio Bay and Matagorda Bay.
- To synthesize the hurricane, the in-house Wind Enhanced Scheme (WES) was used. The WES scheme was originally developed by the UK Meteorological Office based on Holland's model (Holland, 1975).
- The model resolution is 2 km and the bathymetry and land height originates from one minute GEBCO gridded data (http://www.gebco.net/data_and_products/gridded_bathymetry_data

Detailed modelling Rhine branches

Measures:

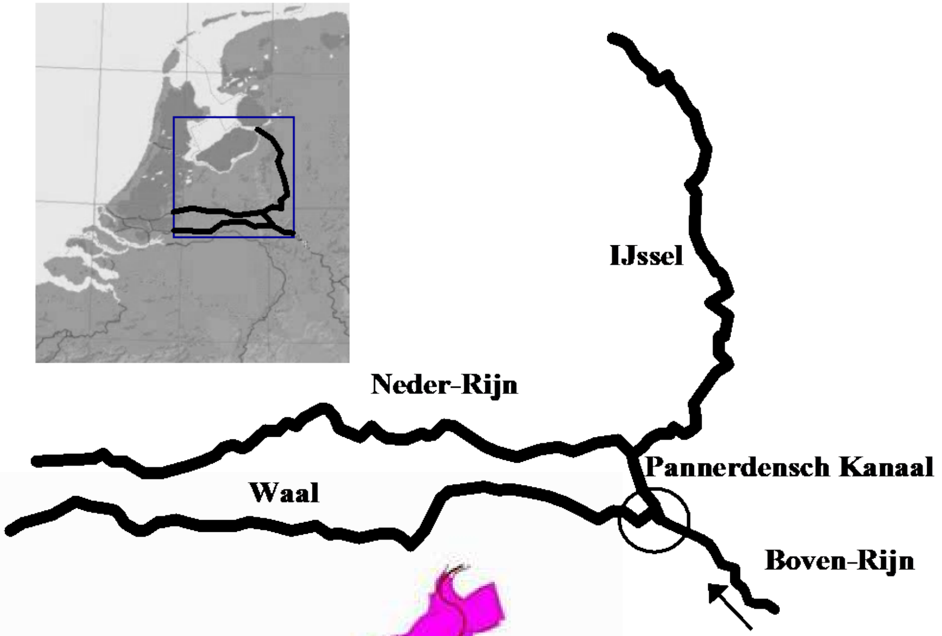
- Dredging
- Channel narrowing by groyne extension
- Measures to correct bend profiles

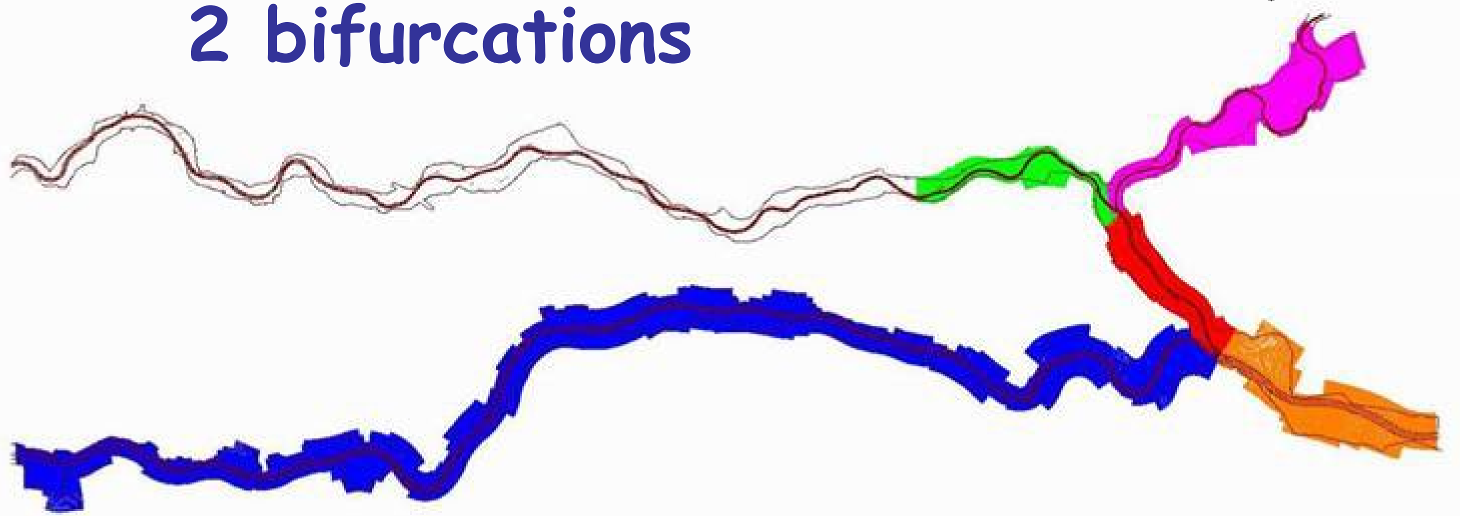


2D numerical model

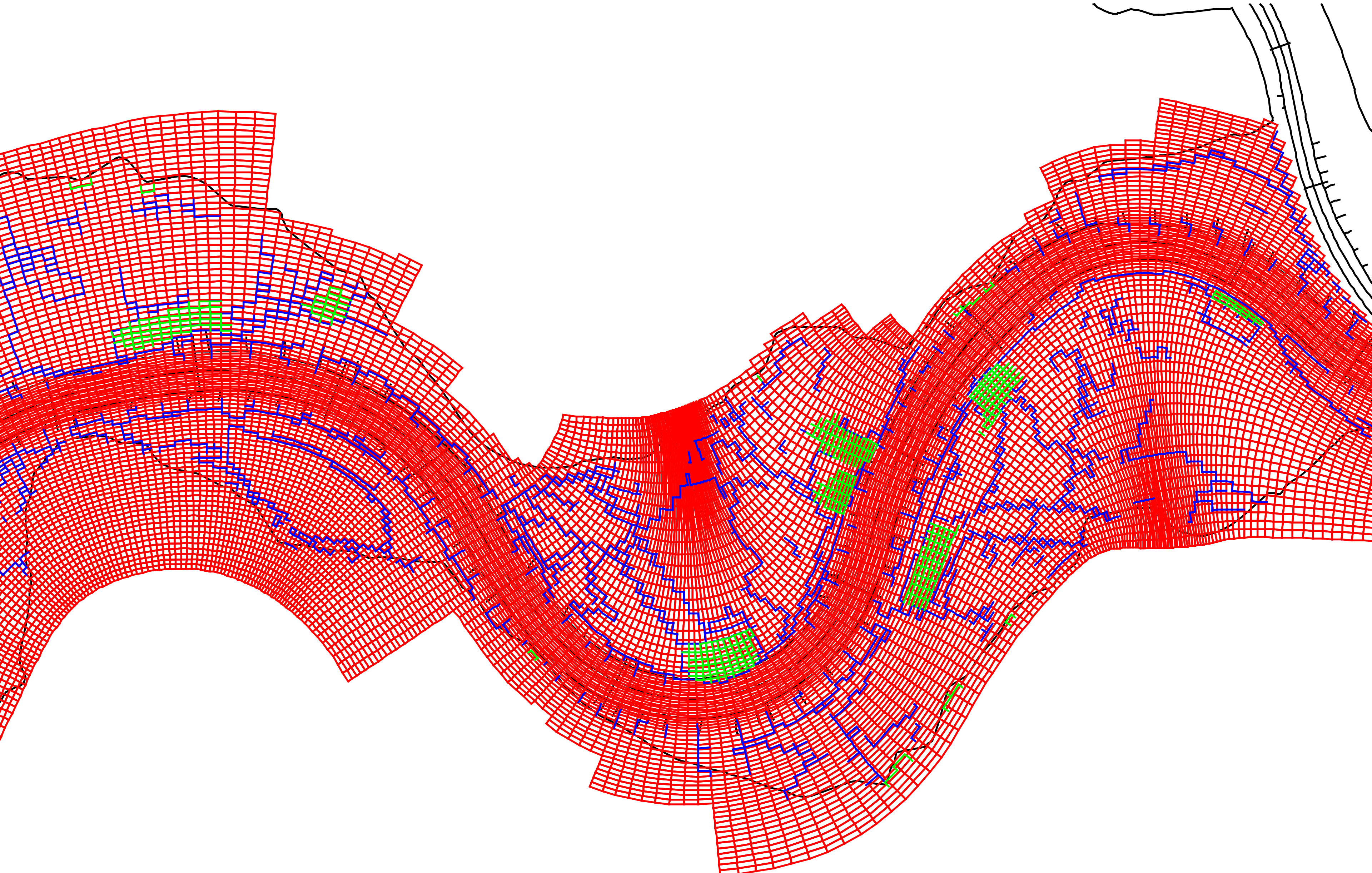
Rhine branches:

2 bifurcations





5 domains, to be extended to Duisburg



Use of 2D numerical model

1. Model construction
2. Hydraulic calibration
3. Morphological calibration:
   i. one-dimensional
   ii. two-dimensional
4. Verification
5. Application

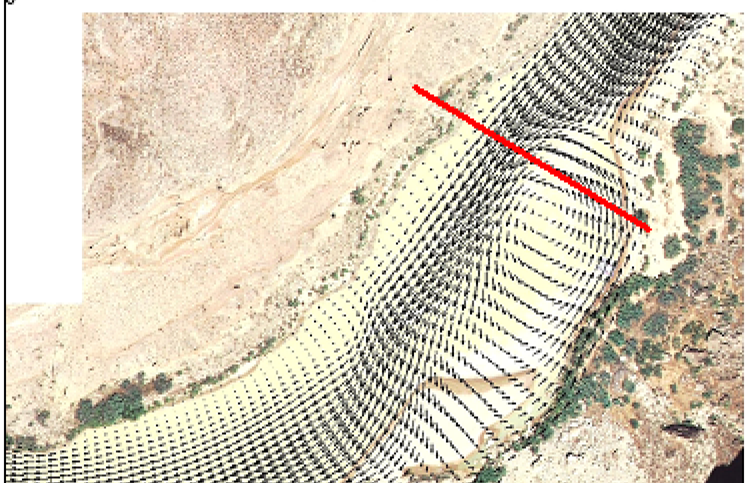
Projects: River Management / Maintenance

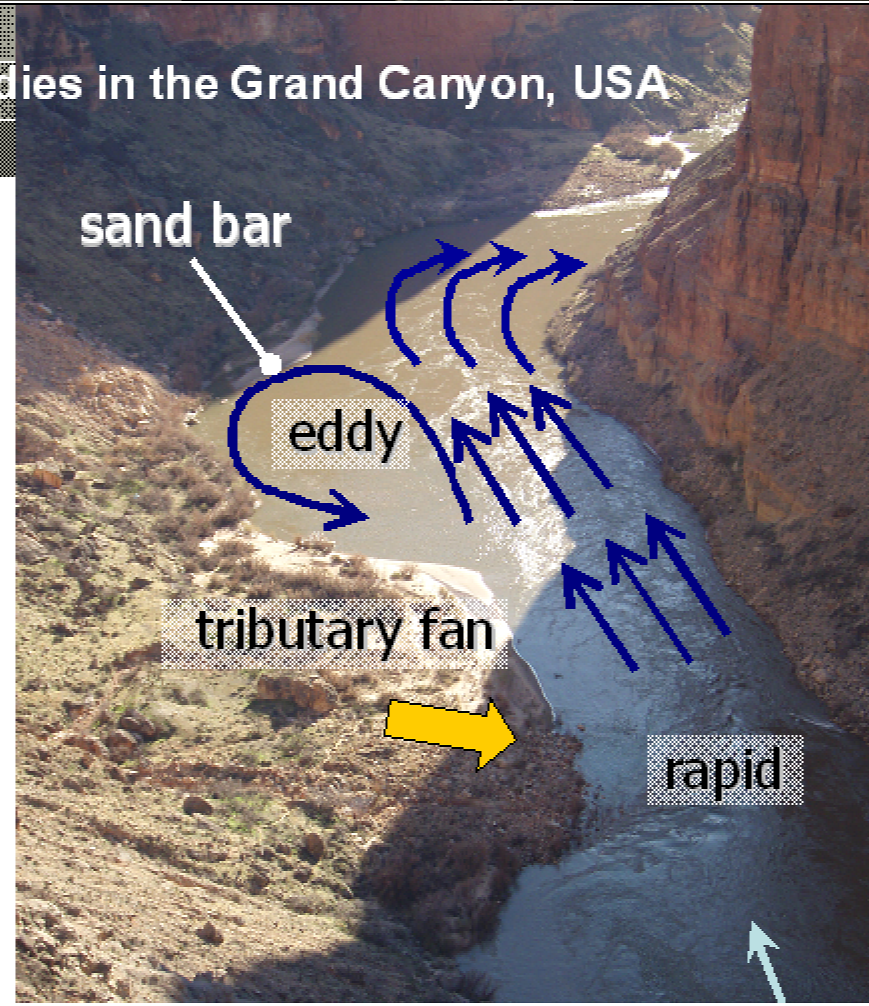
Clients : Rijkswaterstaat / Nat. Power (Japan) / USGS

Research : Geology & Long-Term River Formation / Morphology / Validation

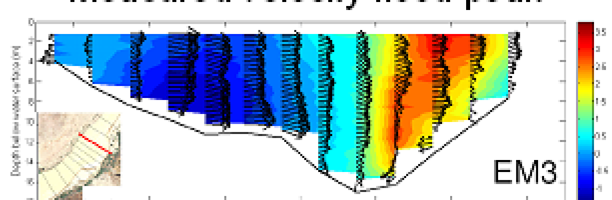
Sand bars in recirculating eddies in the Grand Canyon, USA

- Co-operation with USGS 2007-present: Kees Sloff (Deltares), Scott Wright, Jon Nelson (USGS)
- Validation of Delft3D for 3D turbulent flow and morphology

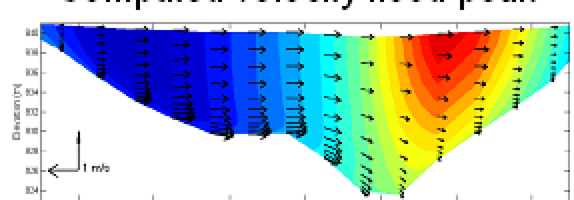




Measured velocity flood peak

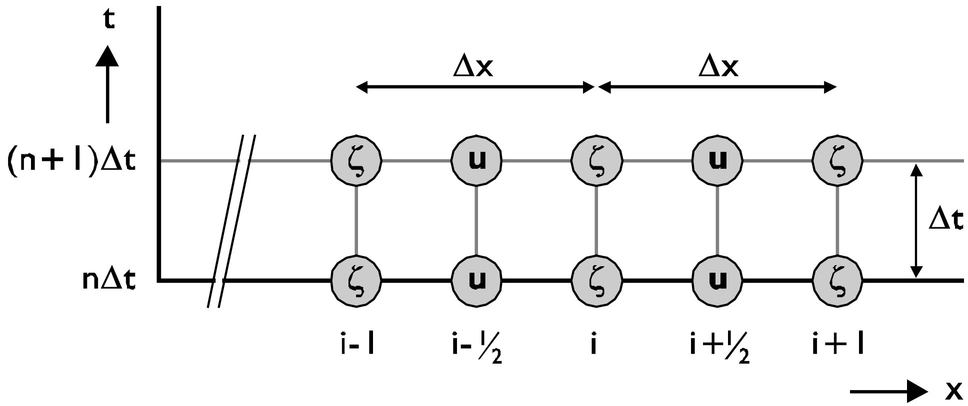


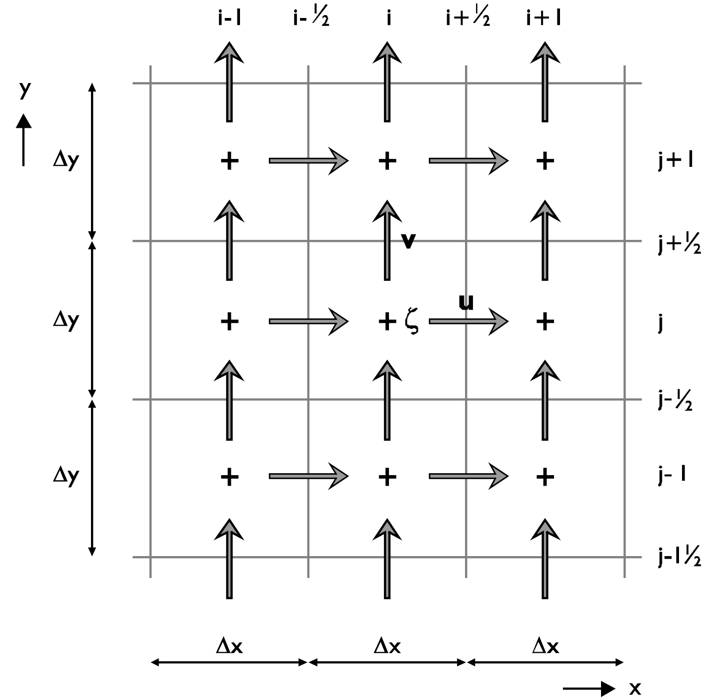
Computed velocity flood peak

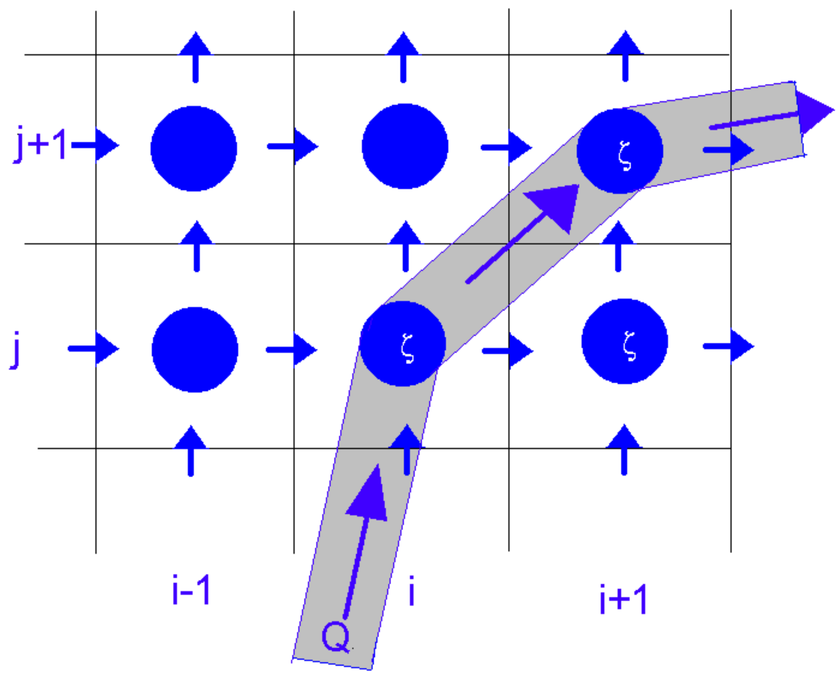


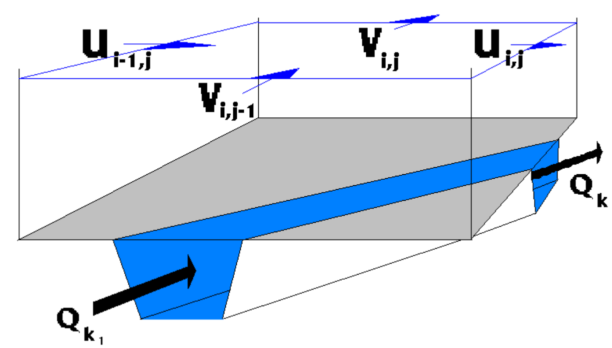
Scott Wright, USGS

Integrated numerical grids



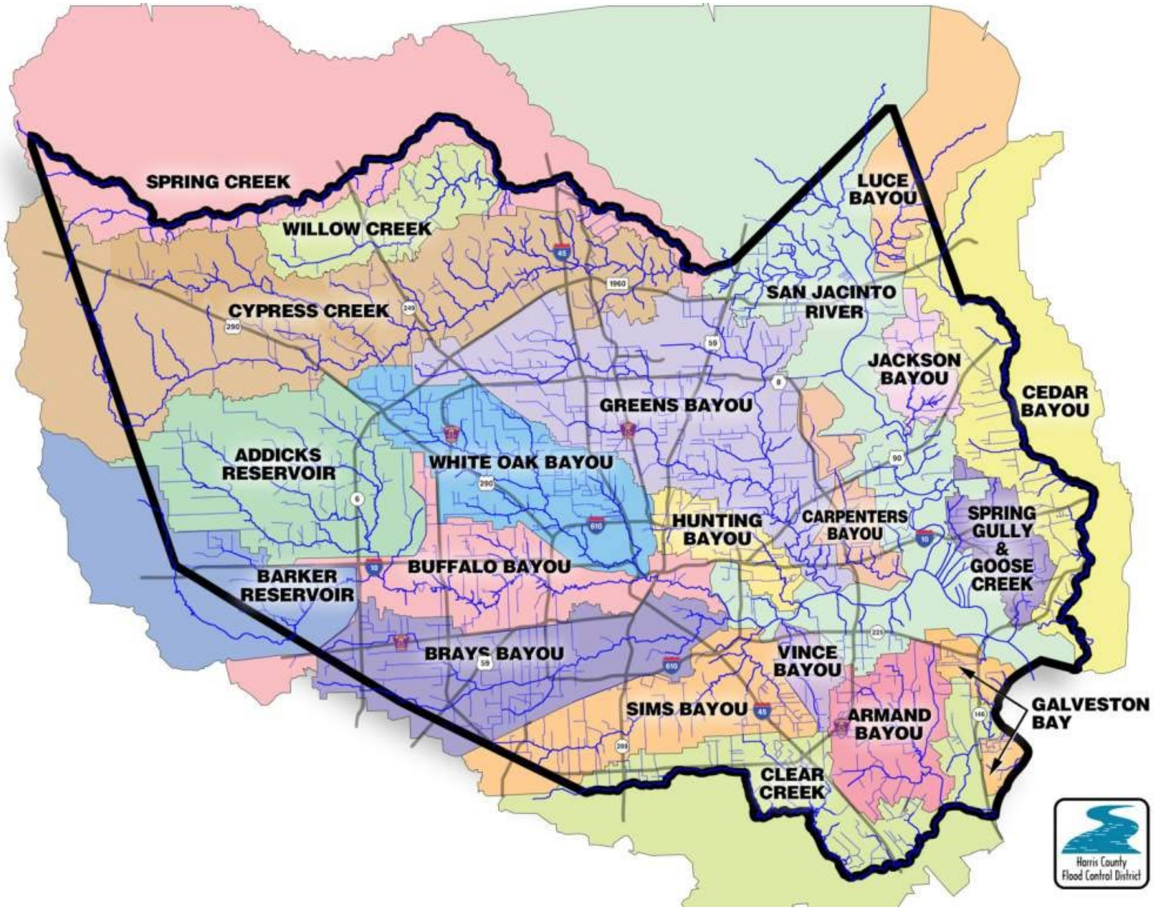




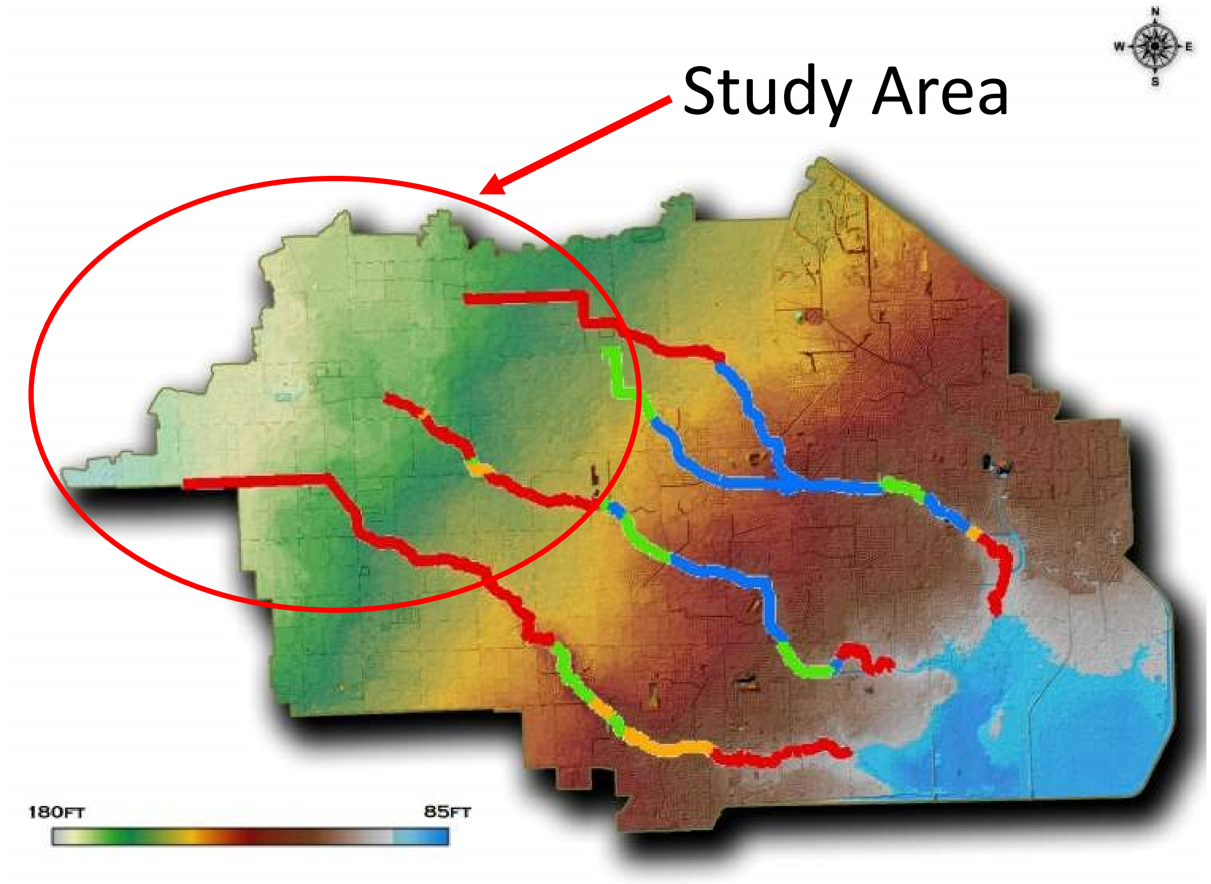


Project ‘Cypress Creek, Texas, USA’

Study area


SPRING CREEK
WILLOW CREEK
CYPRESS CREEK
LUCE BAYOU
SAN JACINTO RIVER
JACKSON BAYOU
CEDAR BAYOU
GREENS BAYOU
ADDICKS RESERVOIR
WHITE OAK BAYOU
HUNTING BAYOU
CARPENTERS BAYOU
SPRING GULLY & GOOSE CREEK
BARKER RESERVOIR
BUFFALO BAYOU
BRAYS BAYOU
VINCE BAYOU
SIMS BAYOU
ARMAND BAYOU
GALVESTON BAY
CLEAR CREEK
Harris County Flood Control District

Study area



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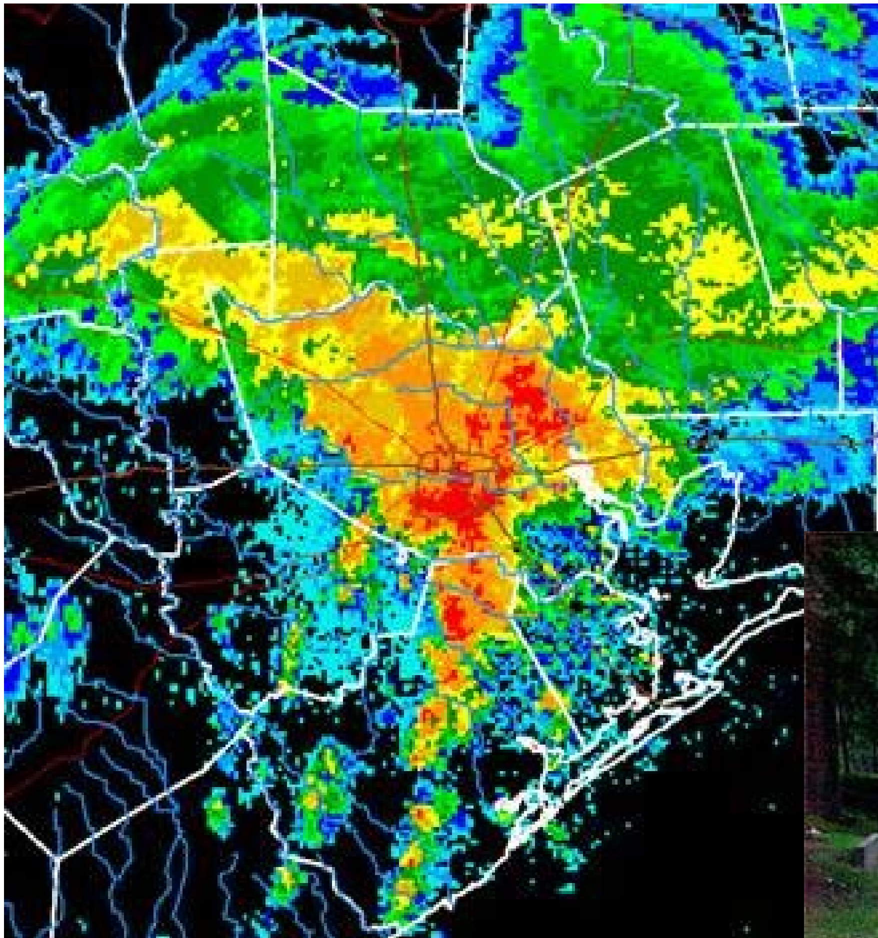
Tropical Storm Allison, 2001



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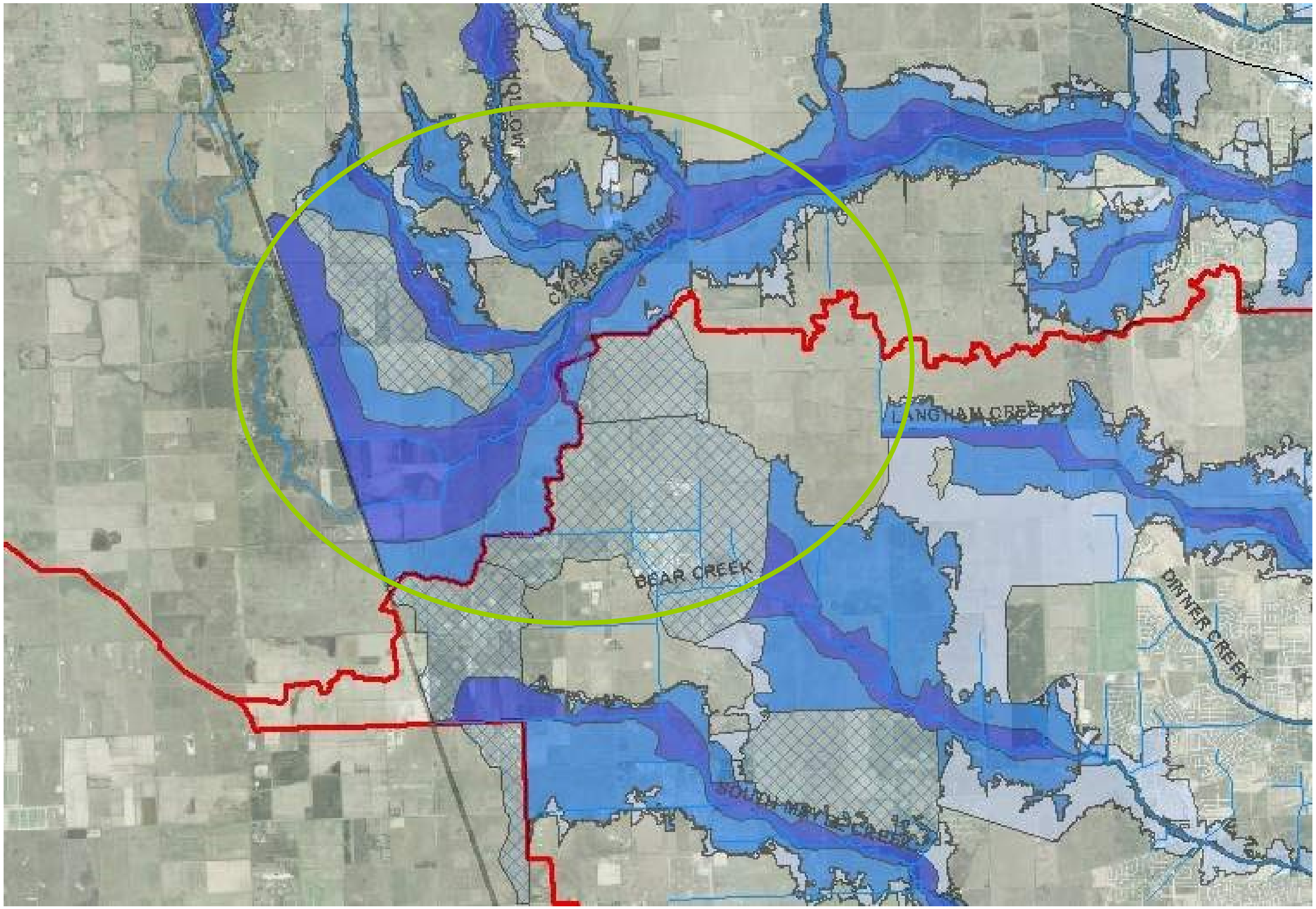
New FEMA Map, based on SOBEK



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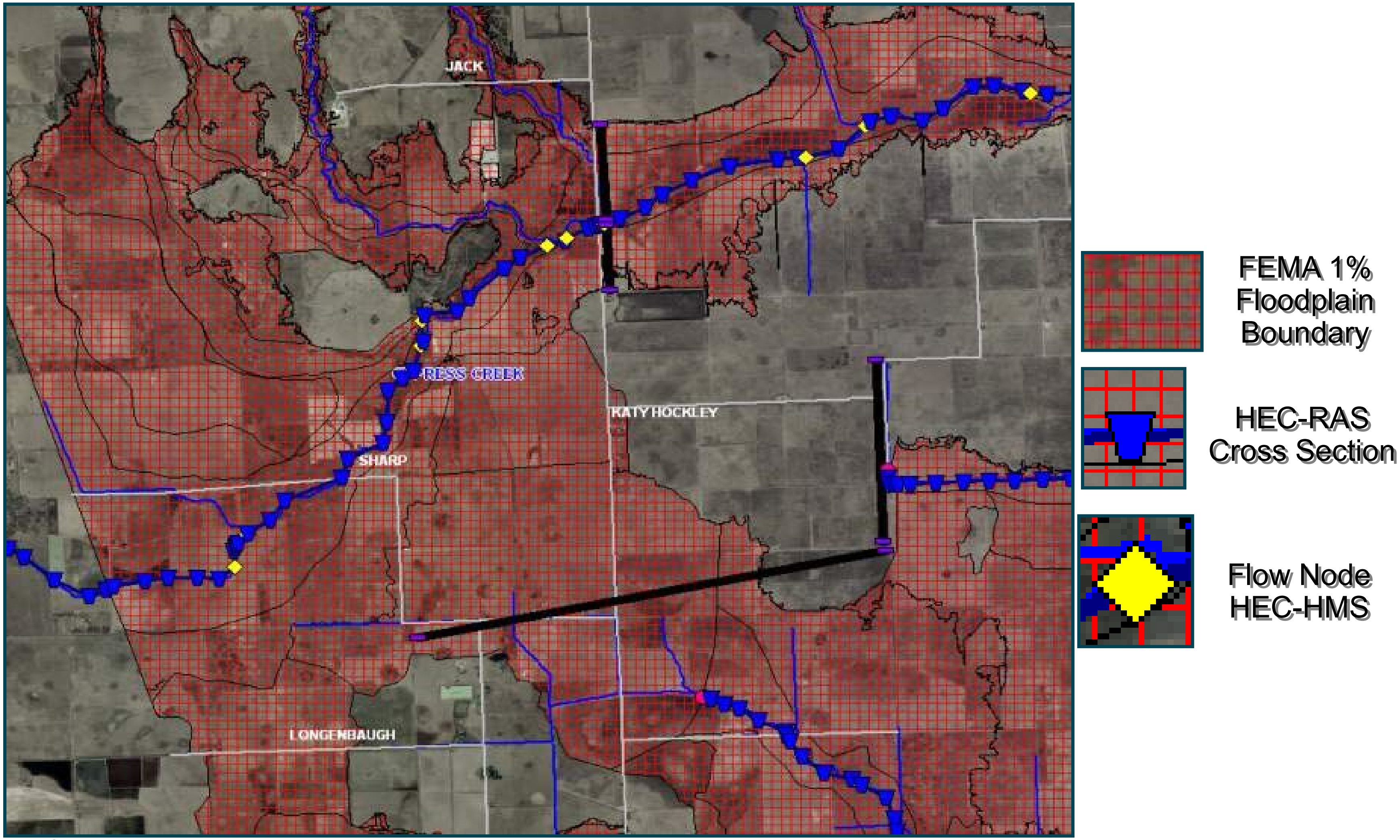
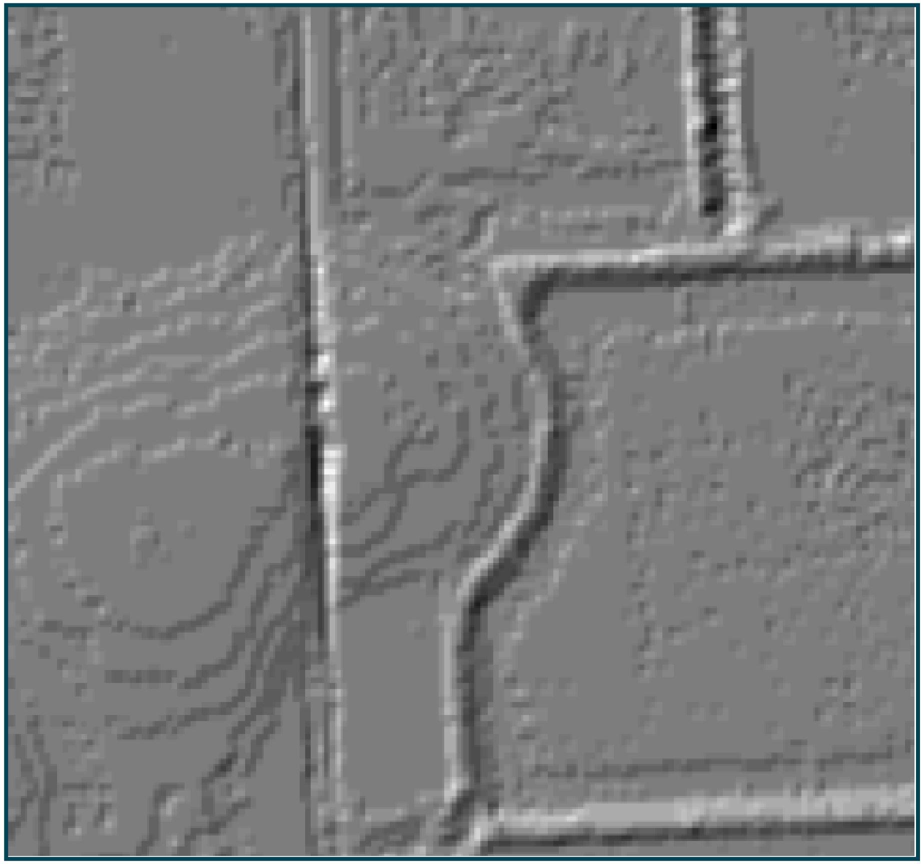
Integrated SOBEK 1D-2D model



Input data: LiDAR data, …

- **Raw 1-ft LiDAR**



Bare Earth 15-ft LiDAR

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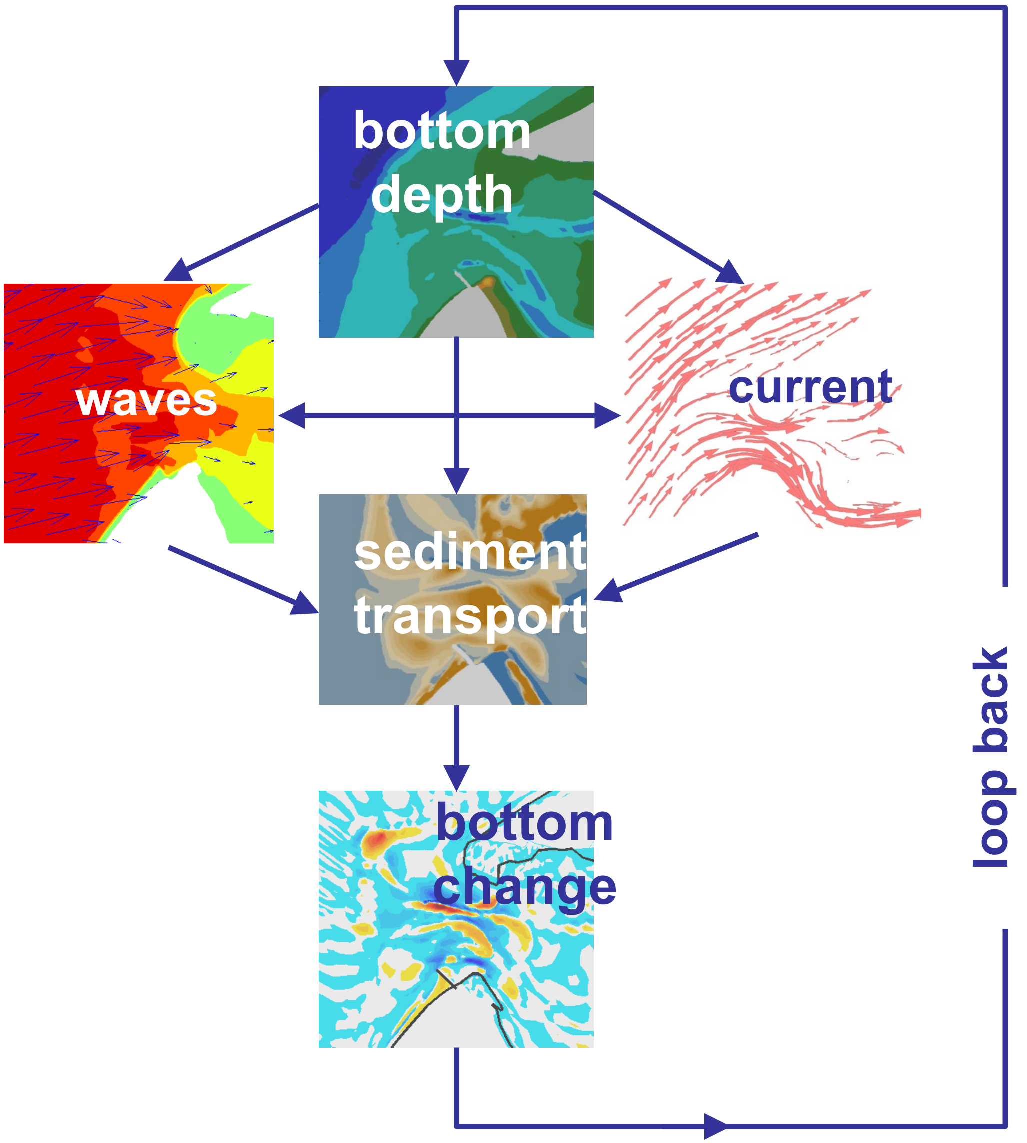
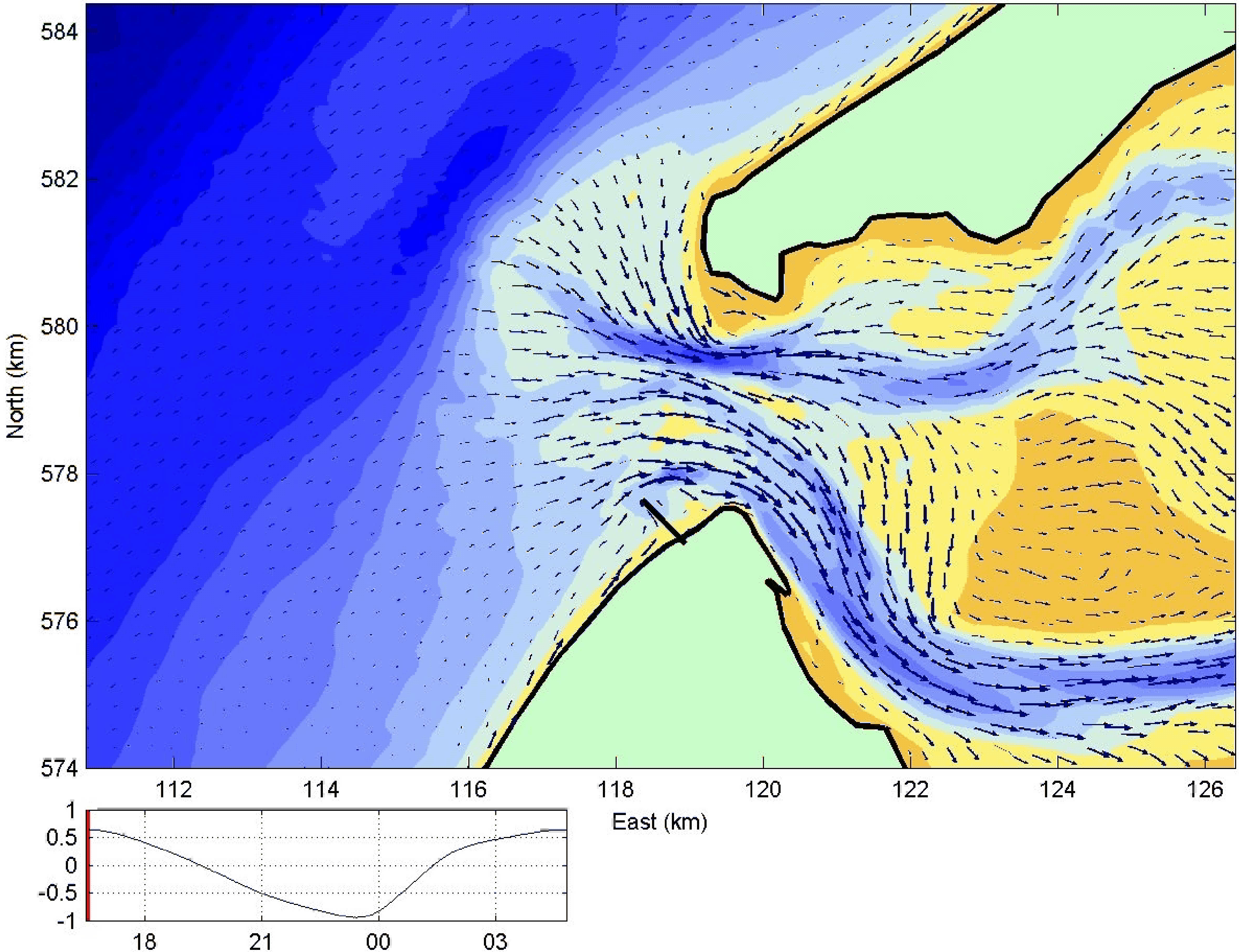
1998 Flooded Structures Summary, Computed vs. Observed

Address	Ponding in Inches		Remarks
	Observed (1)	Computed (2)	
10502 Katy Hockley	8 -inch	9.6 -inch	Finish Floor Unknown
10866 Katy Hockley	14 -inch	15.6 -inch	Finish Floor Unknown
10870 Katy Hockley	22 -inch	22.8 -inch	Finish Floor Unknown
26253 Sharp Rd	3-inch	4.8 -inch	Finish Floor Unknown
26257 Sharp Rd	Unknown	4.0 -inch	Finish Floor Unknown
27010 Sharp Rd	20 -inch	20.4 -inch	Finish Floor Unknown

Process-based, morphological modelling

(Delft3D, XBeach)





584
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574
North (km)
112
114
116
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124
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East (km)
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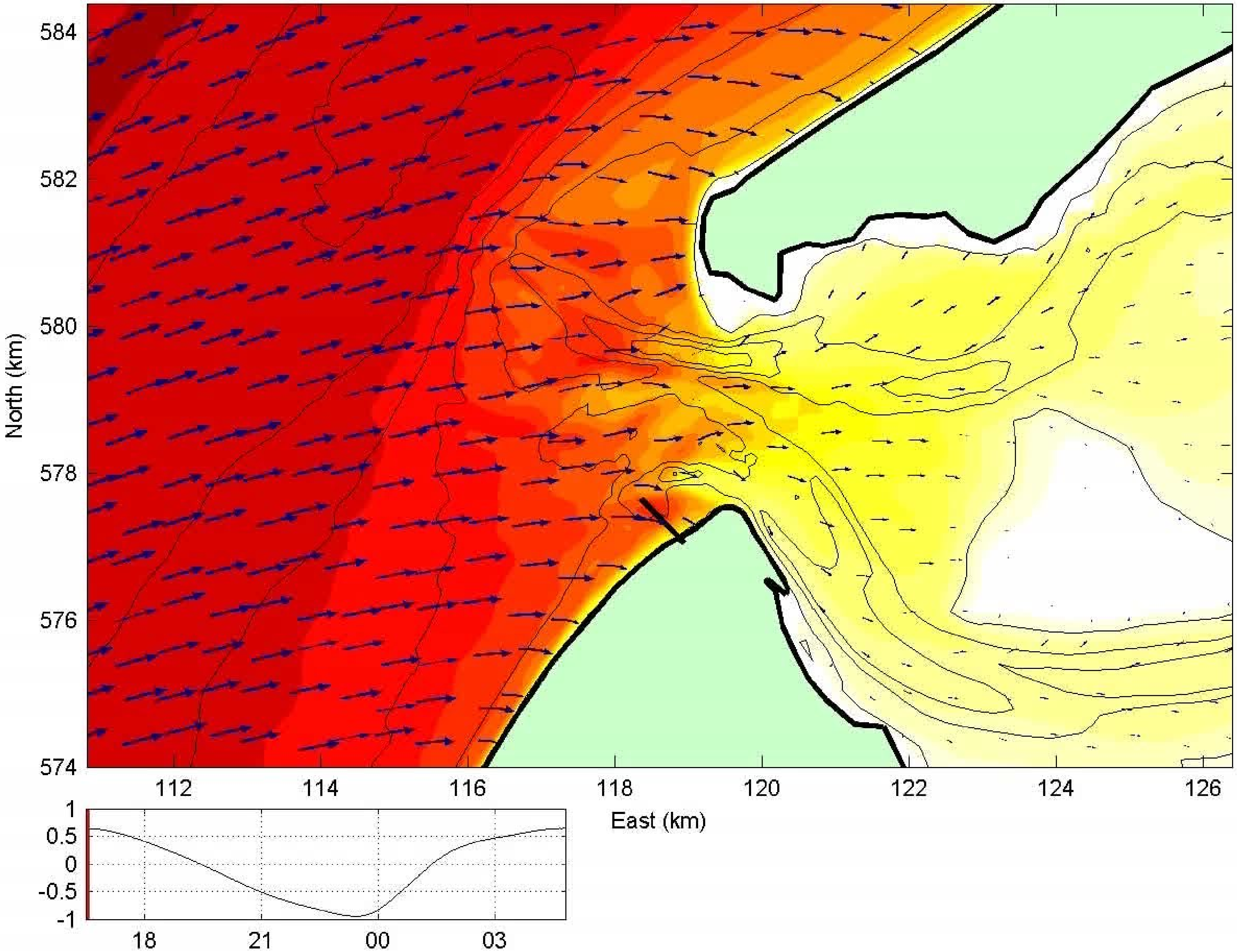

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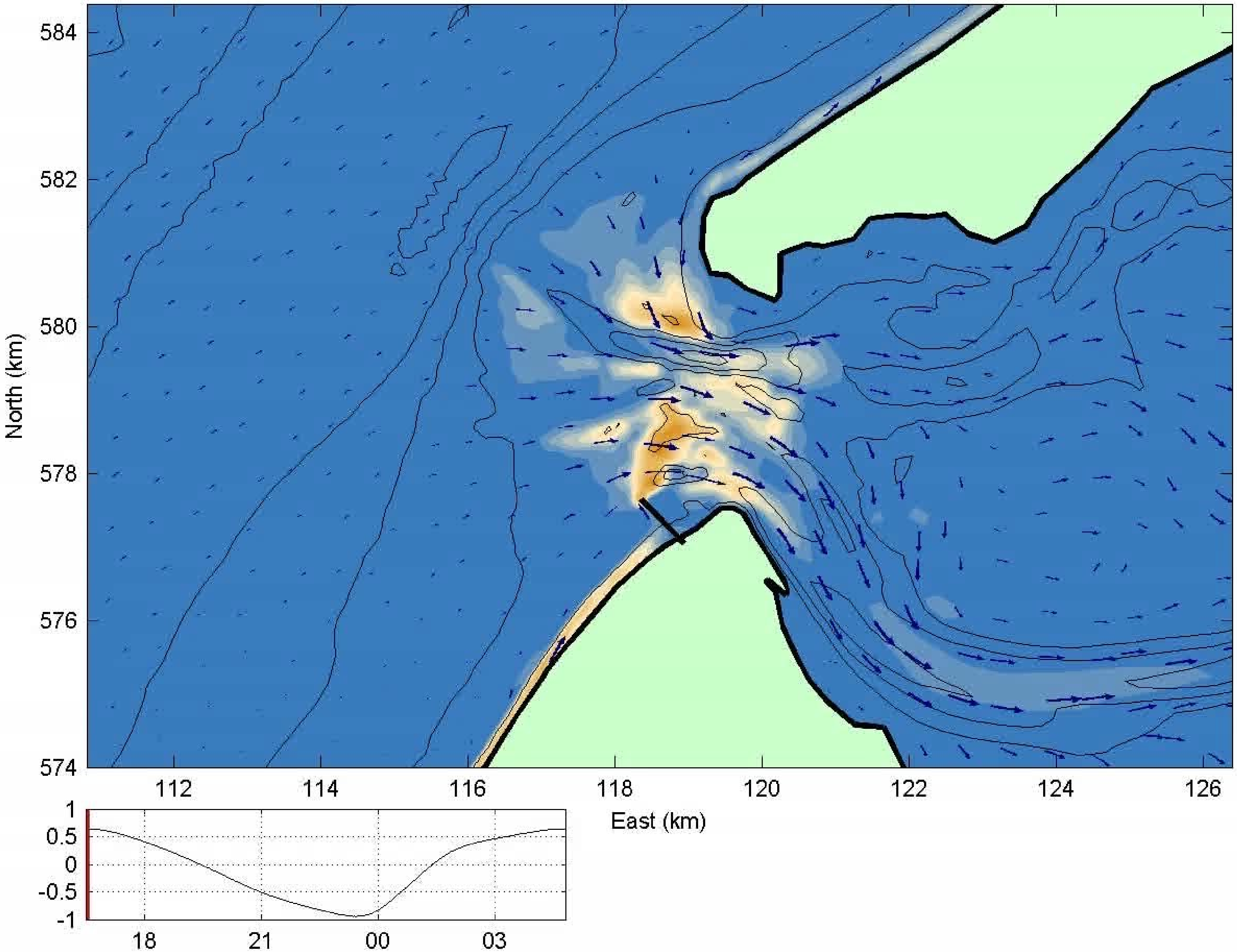

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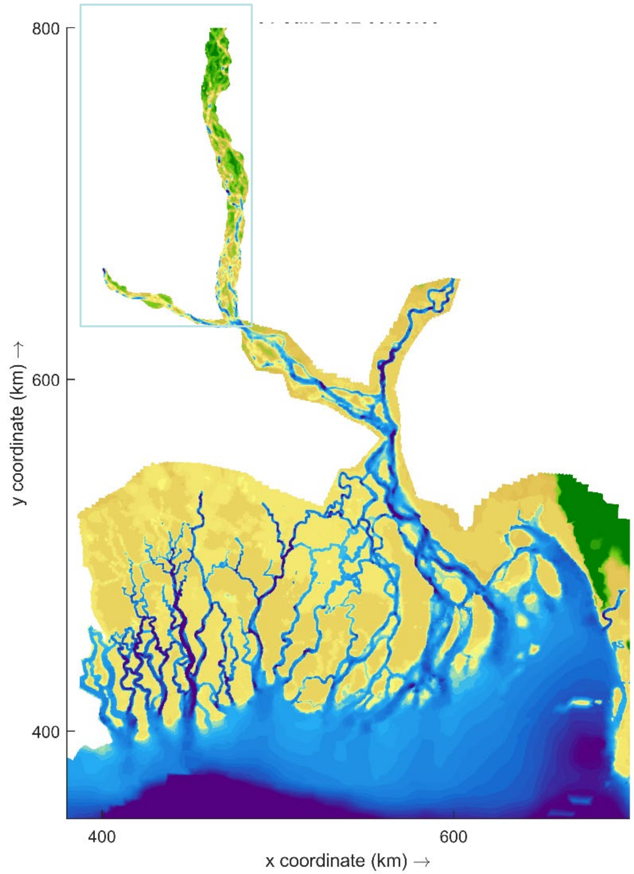
Texel morphology

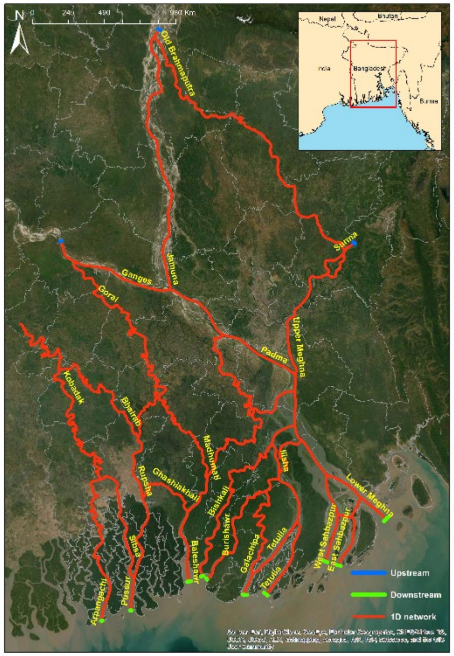
Bangladesh Long-term Monitoring and Modelling Project - CEIP

- Objectives:
  - Large-scale tidal propagation and flow distribution: how do the tidal amplitudes vary through the system and how is this expected to change in the future?
  - Sand and fine sediment distribution: how are different sediment fractions distributed, where are they deposited and how will human intervention and climate change affect this?
  - Pathways for fine sediment: how does the fine sediment make its way through the system and how does it end up in the Sundarbans?
  - Morphology of major channels on decadal scales: can we understand the major morphological changes GBM delta, what processes drives them and how will this change under future scenarios?
  - To provide boundary conditions in terms of large-scale bed elevation change and sediment concentrations to smaller-scale models.

Approach

- Macro-scale 2DH model
  - from Confluence to coast and BoB, optionally including Ganges and Jamuna
  - Resolution from 8km to 500m
  - Coarse but fast
- Macro-scale 1D model
  - Covers major branches
  - Good representation of cross-sections
  - Lean and mean





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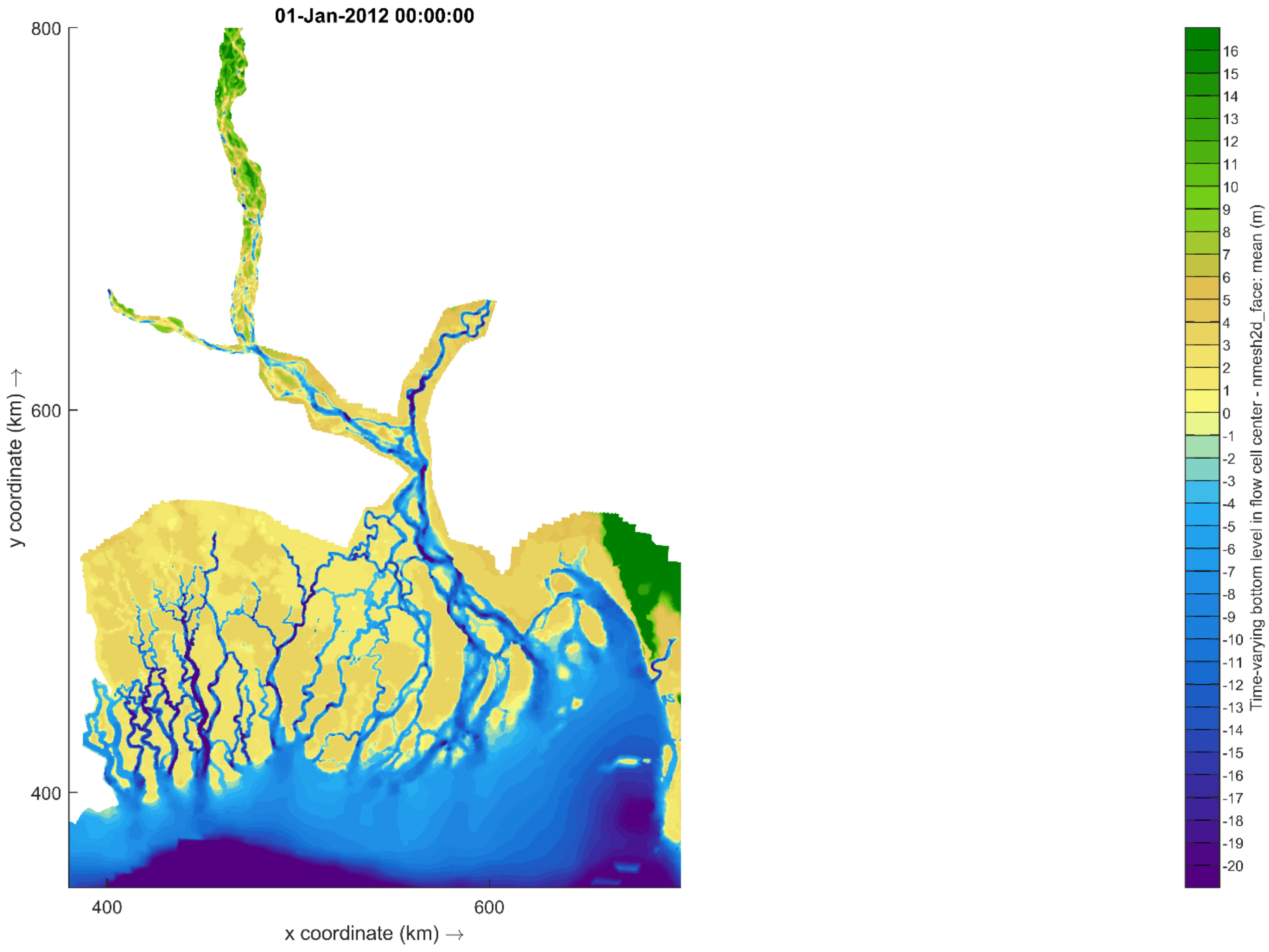
2010



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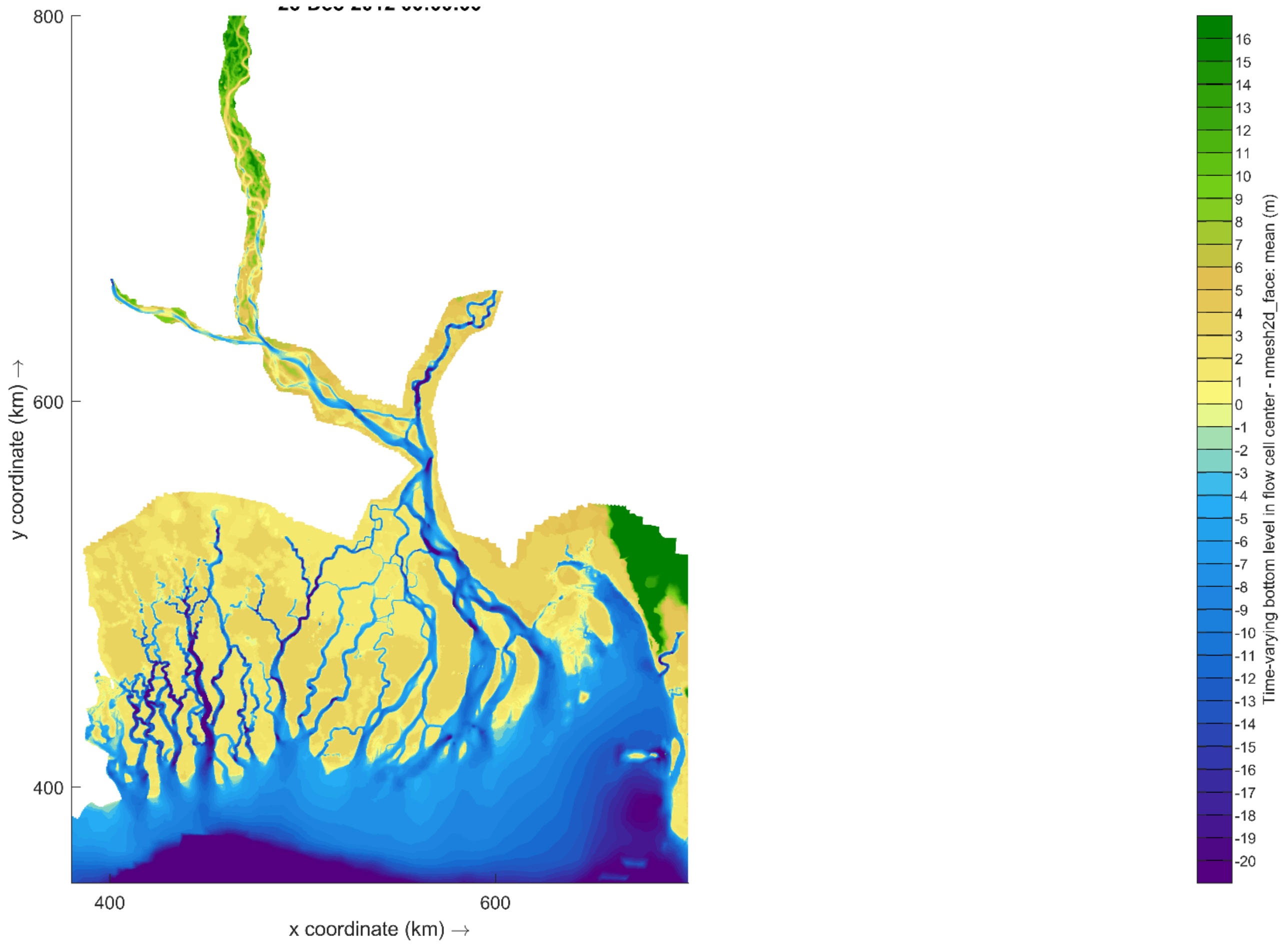
2035?



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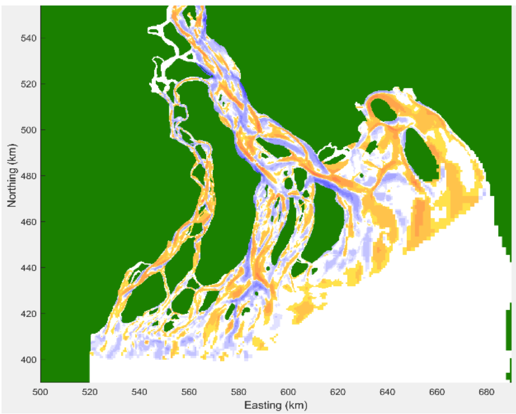
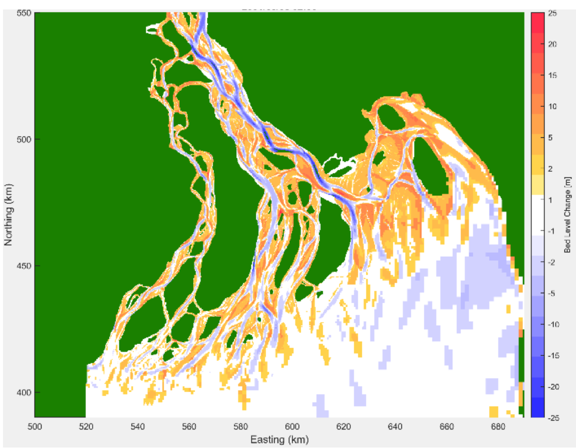
Silt concentration varying through tidal and seasonal cycle

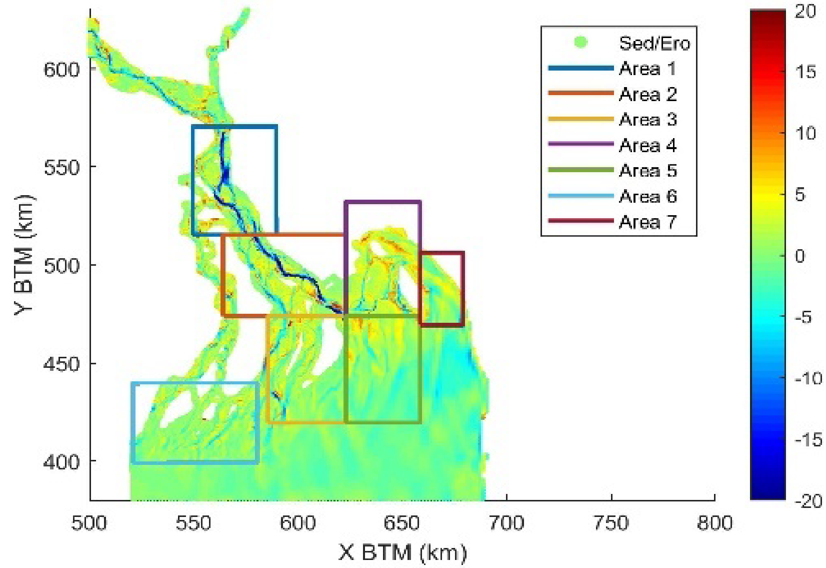
Validation 2000-2009

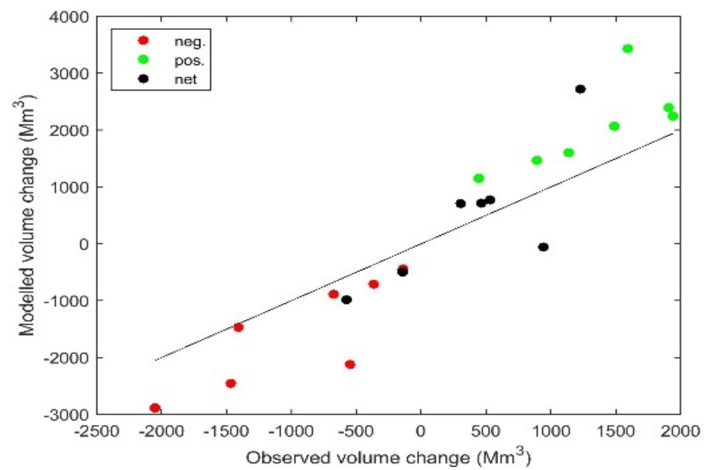


Observed



Modelled





Take home messages

- Go look for examples in your own field of interest
- Try to find peer-reviewed publications of the models you consider, don't believe the brochures
- Don't believe the prettiest picture
- Always assume that the model is wrong until proven otherwise

When to use which model

- 1D
  - narrow channels, L/W big
  - networks of channels easily represented
  - fast
  - good representation of
- 2DH
  - floodplains
  - important horizontal variations in
    - bathymetry
    - forcing
    - geometry

When to use which model

- Quasi-3D
  - as in 2DH but with modifications to account for spiral flow (river bends) or return flow (surf zone)
  - same computational effort as 2DH (1 layer)
- 3D hydrostatic
  - to resolve variations over the vertical, e.g. for water quality
    - O2, nutrients, sediment concentration, ...
  - in case of vertical variations of forcing, e.g. by density differences (due to salinity, temperature)
  - especially important when there is stratification, e.g. in deep lakes
- 3D nonhydrostatic
  - for local problems such as flow around bridge piles

Tsunamis

- Look up examples of studies
- Are shallow water equations used?
- If not, why not?
- Why are they so destructive?

Review papers

PHILOSOPHICAL TRANSACTIONS A

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Review

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One contribution of 14 to a theme issue 'Tsunamis: bridging science, engineering and society'.

Subject Areas:
mathematical modelling, wave motion

New computational methods in tsunami science

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Tsunamis are rare events with severe consequences. This generates a high demand on accurate simulation results for planning and risk assessment purposes because of the low availability of actual data from historic events. On the other hand, validation of simulation tools becomes very difficult with such a low amount of real-world data. Tsunami phenomena involve a large span of spatial and temporal scales—from ocean basin scales of $\mathcal{O}(10^7)$ m to local coastal wave interactions of $\mathcal{O}(10^2)$ m or even $\mathcal{O}(10^1)$ m, or from resonating wave phenomena with durations of $\mathcal{O}(10^5)$ s to rupture with time periods of $\mathcal{O}(10^1)$ s. The scale gap of five orders of magnitude in

Application papers

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 116, F01006, doi:10.1029/2010JF001797, 2011

Process-based modeling of tsunami inundation and sediment transport

Alex Apotsos,[1] Guy Gelfenbaum,[1] and Bruce Jaffe[2]

Received 8 June 2010; revised 15 November 2010; accepted 30 November 2010; published 10 February 2011.

[1] The infrequent and unpredictable nature of tsunamis precludes the use of field experiments to measure the hydrodynamic and sediment transport processes that occur. Instead, these processes are often approximated from laboratory, numerical, and theoretical studies or inferred from observations of the resultant sediment deposits. Here Delft3D, a three-dimensional numerical model, is used to simulate the inundation and sediment transport of a tsunami similar in magnitude to the 26 December 2004 Indian Ocean tsunami over one measured and three idealized morphologies. The model is first shown to match well the observations taken at Kuala Meurisi, Sumatra, and then used to examine in detail the processes that occur during the tsunami. The model predicts that at a given cross-shore location the onshore flow accelerates rapidly to a maximum as the wavefront passes, and then gradually decelerates before reversing direction and flowing

Types of models

- NSWE (COMCOT, Delft3D, XBeach)
- Boussinesq (FUNWAVE, COULWAVE)
- Nonhydrostatic (SWASH, XBeach-nonh)
- Detailed CFD models (OpenFoam, SPH)

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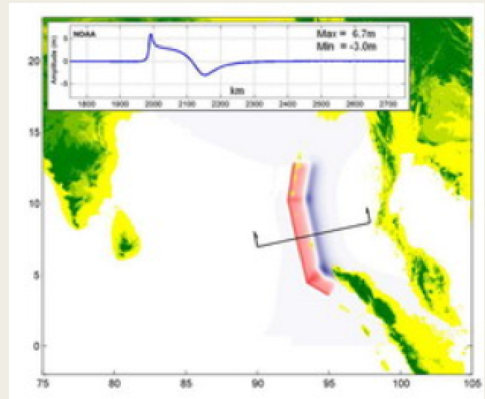
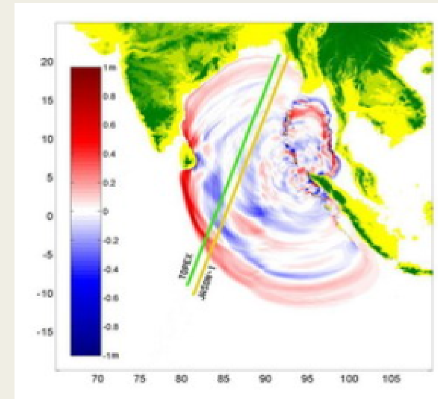
COMCOT

- Introduction
- Background
- Fault Model
- Applications
- Updates
- Downloads
- Links

COMCOT: A Tsunami Modeling Package



Tsunami Generation



Tsunami Propagation

COMCOT (Cornell Multi-grid Coupled Tsunami Model) is a tsunami modeling package, capable of simulating the entire lifespan of a tsunami, from its generation, propagation and runup/rundown in coastal regions.

Waves can be generated via incident wave maker, fault model, landslide, or even customized profile. Flexible nested grid setup allows for the balance between accuracy and efficiency.

The model has been used to investigate several historical tsunami events, such as the 1960 Chilean tsunami, the 1992 Flores Islands (Indonesia) tsunami (*Liu et al., 1994; Liu et al., 1995*), the 2003 Algeria Tsunami (*Wang and Liu, 2005*) and more recently the 2004 Indian Ocean tsunami (*Wang and Liu, 2006*).

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COMCOT: Background Theory

- **Governing Equations**

COMCOT was developed based on Shallow Water Equations (SWE) in Spherical Coordinates (*Eq.01*) and Cartesian Coordinates (*Eq.02*). In the equations, ζ denotes free surface elevation; P and Q are volume flux in x and y direction ($P=hu$, $Q=hv$); φ and ψ stand for longitude and latitude, respectively.

$$\frac{\partial \zeta}{\partial t} + \frac{1}{R\cos\varphi}\left[\frac{\partial P}{\partial \psi} + \frac{\partial}{\partial \varphi}(\cos\varphi Q)\right] = 0$$

$$\frac{\partial P}{\partial t} + \frac{gh}{R\cos\varphi}\frac{\partial \zeta}{\partial \psi} - fQ = 0$$

$$\frac{\partial Q}{\partial t} + \frac{gh}{R}\frac{\partial \zeta}{\partial \varphi} + fP = 0$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0$$

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x}\left(\frac{P^2}{H}\right) + \frac{\partial}{\partial y}\left(\frac{PQ}{H}\right) + gH\frac{\partial \zeta}{\partial x} + \frac{\tau_x H}{\rho} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{PQ}{H}\right) + \frac{\partial}{\partial y}\left(\frac{Q^2}{H}\right) + gH\frac{\partial \zeta}{\partial y} + \frac{\tau_y H}{\rho} = 0$$

Eq.01 SWE in Spherical Coord. *Eq.02 SWE in Cartesian Coord.*

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XBeach 1755 Lisbon tsunami