3.1 MAIN CONCEPTS AND DEFINITIONS

The basic hydraulic principles applied in water transport and distribution practice emerge from three main assumptions:

1. The system is filled with water under pressure,
2. that water is incompressible,
3. that water has a steady and uniform flow.

In addition, it is assumed that the deformation of the system boundaries is negligible, meaning that the water flows through a non-elastic system.1

**Steady flow**

Flow $Q$ (m$^3$/s) through a pipe cross-section of area $A$ (m$^2$) is determined as $Q = v \times A$, where $v$ (m/s) is the mean velocity in the cross-section. This flow is *steady* if the mean velocity remains constant over a period of time $\Delta t$.

**Uniform flow**

If the mean velocities of two consecutive cross-sections are equal at a particular moment, the flow is *uniform*.

The earlier definitions written in the form of equations for two close moments, $t_1$ and $t_2$, and in the pipe cross-sections 1 and 2 (Figure 3.1) yield:

\[
\frac{v_1^{(t_1)}}{v_2^{(t_1)}} = \frac{v_1^{(t_2)}}{v_2^{(t_2)}} \quad (3.1)
\]

for a steady flow, and:

\[
\frac{v_1^{(t_1)}}{v_2^{(t_1)}} = \frac{v_1^{(t_2)}}{v_2^{(t_2)}} \quad (3.2)
\]

for a uniform flow.

A steady flow in a pipe with a constant diameter is at the same time uniform. Thus:

\[
\frac{v_1^{(t_1)}}{v_2^{(t_1)}} = \frac{v_1^{(t_2)}}{v_2^{(t_2)}} = \frac{v_1^{(t_1)}}{v_2^{(t_1)}} = \frac{v_1^{(t_2)}}{v_2^{(t_2)}} \quad (3.3)
\]

---

1 The foundations of steady state hydraulics are described in detail in various references of Fluid Mechanics and Engineering Hydraulics. See for instance Streeter and Wylie (1985).
Transient flow

The earlier simplifications help to describe the general hydraulic behaviour of water distribution systems assuming that the time interval between \( t_1 \) and \( t_2 \) is sufficiently short. Relatively slow changes of boundary conditions during regular operation of these systems make \( \Delta t \) of a few minutes acceptably short for the assumptions introduced earlier. At the same time, this interval is long enough to simulate changes in pump operation, levels in reservoirs, diurnal demand patterns, etc., without handling unnecessarily large amounts of data. If there is a sudden change in operation, for instance a situation caused by pump failure or valve closure, transitional flow conditions occur in which the assumptions of the steady and uniform flow are no longer valid. To be able to describe these phenomena in a mathematically accurate way, a more complex approach elaborated in the theory of transient flows would be required, which is not discussed in this book. The reference literature on this topic includes Larock et al. (2000).

3.1.1 Conservation laws

The conservation laws of mass, energy and momentum are three fundamental laws related to fluid flow. These laws state:

1. **The Mass Conservation Law**
   Mass \( m \) (kg) can neither be created nor destroyed; any mass that enters a system must either accumulate in that system or leave it.

2. **The Energy Conservation Law**
   Energy \( E \) (J) can neither be created nor destroyed; it can only be transformed into another form.
3 The Momentum Conservation Law

The sum of external forces acting on a fluid system equals the change of the momentum rate \(M\) (N) of that system.

The conservation laws are translated into practice through the application of three equations, respectively:

1. The Continuity Equation.
2. The Energy Equation.
3. The Momentum Equation.

**Continuity Equation**

The Continuity Equation is used when balancing the volumes and flows in distribution networks. Assuming that water is an incompressible fluid, i.e. with a mass density \(\rho = m/V = \text{const}\), the Mass Conservation Law can be applied to volumes. In this situation, the following is valid for tanks (see Figure 3.2):

\[
Q_{\text{imp}} = Q_{\text{out}} \pm \frac{\Delta V}{\Delta t}
\]  \hspace{1cm} (3.4)

where \(\Delta V/\Delta t\) represents the change in volume \(V\) (m\(^3\)) within a time interval \(\Delta t\) (s). Thus, the difference between the input- and output-flow from a tank is the volume that is:

1. accumulated in the tank if \(Q_{\text{out}} < Q_{\text{imp}}\) (sign + in Equation 3.4),
2. withdrawn from the tank if \(Q_{\text{out}} > Q_{\text{imp}}\) (sign -).

Applied at node \(n\) that connects \(j\) pipes, the Continuity Equation can be written as:

\[
\sum_{i=1}^{j} Q_i - Q_n = 0
\]  \hspace{1cm} (3.5)

where \(Q_n\) represents the nodal discharge. An example of three pipes and a discharge point is shown in Figure 3.3.

**Energy Equation**

The Energy Equation establishes the energy balance between any two cross-sections of a pipe:

\[
E_1 = E_2 \pm \Delta E
\]  \hspace{1cm} (3.6)

![Figure 3.2. The Continuity Equation validity in tanks.](image-url)
where $\Delta E$ is the amount of transformed energy between cross-sections 1 and 2. It is usually the energy lost from the system (the sign $+$ in Equation 3.6), but may also be added to it by pumping of water (sign $-$).

**Momentum Equation**

The Momentum Equation (in some literature also known as the Dynamic Equation) describes the pipe resistance to dynamic forces caused by the pressurised flow. For incompressible fluids, momentum $M$ (N) carried across a pipe section is defined as:

$$M = \rho Q v$$  \hspace{1cm} (3.7)

where $\rho$ (kg/m$^3$) represents the mass density of water, $Q$(m$^3$/s) is the pipe flow, $v$ (m/s) is the mean velocity. Other forces in the equilibrium are (see Figure 3.4):

1. Hydrostatic force $F_h$ (N) caused by fluid pressure $p$ (N/m$^2$ or Pa);
   $$F_h = p \times A.$$
2. Weight $w$ (N) of the considered fluid volume (only acts in a vertical direction).
3. Force $F$ (N) of the solid surface acting on the fluid.

The Momentum Equation as written for a horizontal direction would state:

$$\rho Q v_1 - \rho Q v_2 \cos \varphi = - p_1 A_1 + p_2 A_2 \cos \varphi + F_x$$  \hspace{1cm} (3.8)

whereas in a vertical direction:

$$\rho Q v_2 \sin \varphi = - p_2 A_2 \sin \varphi + w + F_y$$  \hspace{1cm} (3.9)

**Pipe thrust**

The forces of the water acting on the pipe bend are the same, i.e. $F_x$ and $F_y$, but with an opposite direction i.e. a negative sign, in which case the
total force, known as the *pipe thrust* will be:

\[
F = \sqrt{F_x^2 + F_y^2}
\]  
(3.10)

The Momentum Equation is applied in calculations for the additional strengthening of pipes, in locations where the flow needs to be diverted. The results are used for the design of concrete structures required for anchoring of pipe bends and elbows.

**PROBLEM 3.1**

A velocity of 1.2 m/s has been measured in a pipe of diameter \(D = 600 \text{ mm}\). Calculate the pipe flow.

*Answer:*

The cross-section of the pipe is:

\[
A = \frac{D^2\pi}{4} = \frac{0.6^2 \times 3.14}{4} = 0.2827 \text{ m}^2
\]

which yields the flow of:

\[
D = vA = 1.2 \times 0.2827 = 0.339 \text{ m}^3/\text{s} \approx 340 \text{ l/s}
\]

**PROBLEM 3.2**

A circular tank with a diameter at the bottom of \(D = 20 \text{ m}\) and with vertical walls has been filled with a flow of 240 m\(^3\)/h. What will be the increase of the tank depth after 15 minutes, assuming a constant flow during this period of time?

*Answer:*

The tank cross-section area is:

\[
A = \frac{D^2\pi}{4} = \frac{20^2 \times 3.14}{4} = 314.16 \text{ m}^2
\]
The flow of 240 m³/h fills the tank with an additional 60 m³ after 15 minutes, which is going to increase the tank depth by a further \(\frac{60}{314.16} = 0.19 \text{ m} \approx 20 \text{ cm}\).

**PROBLEM 3.3**

For a pipe bend of 45° and a continuous diameter of \(D = 300 \text{ mm}\), calculate the pipe thrust if the water pressure in the bend is 100 kPa at a measured flow rate of 26 l/s. The weight of the fluid can be neglected. The mass density of the water equals \(\rho = 1000 \text{ kg/m}^3\).

**Answer:**

From Figure 3.4, for a continuous pipe diameter:

\[
A_1 = A_2 = \frac{D^2 \pi}{4} = \frac{0.3^2 \times 3.14}{4} = 0.07 \text{ m}^2
\]

Consequently, the flow velocity in the bend can be calculated as:

\[
v_1 = v_2 = \frac{Q}{A} = \frac{0.026}{0.07} = 0.37 \text{ m/s}
\]

Furthermore, for the angle \(\phi = 45^\circ\), \(\sin \phi = \cos \phi = 0.71\). Assuming also that \(p_1 = p_2 = 100 \text{ kPa}\) (or 100,000 N/m²), the thrust force in the X-direction becomes:

\[
-F_x = 0.29 \times (pA + \rho Qv) = 0.29 \times (100,000 \times 0.07 + 1000 \times 0.026 \times 0.37) \approx 2030 \text{ N} = 2 \text{ kN}
\]

while in the Y-direction:

\[-F_y = 0.71 \times (pA + \rho Qv) = 5 \text{ kN}\]

The total force will therefore be:

\[
F = \sqrt{2^2 + 5^2} \approx 5.4 \text{ kN}
\]

The calculation shows that the impact of water pressure is much more significant than the one of the flow/velocity.

**Self-study:**

Spreadsheet lesson A5.1.1 (Appendix 5)

### 3.1.2 Energy and hydraulic grade lines

The energy balance in Equation 3.6 stands for total energies in two cross-sections of a pipe. The total energy in each cross-section comprises three components, which is generally written as:

\[
E_{\text{tot}} = mgZ + \frac{p}{\rho} + \frac{mv^2}{2}
\]  

(3.11)
expressed in J or more commonly in kWh. Written per unit weight, the equation looks as follows:

\[ E_{\text{tot}} = Z + \frac{p}{\rho g} + \frac{v^2}{2g} \]  \hspace{1cm} (3.12)

where the energy obtained will be expressed in metres water column (mwc). Parameter \( g \) in both these equations stands for gravity (9.81 m/s²).

**Potential energy**

The first term in Equations 3.11 and 3.12 determines the *potential energy*, which is entirely dependant on the elevation of the mass/volume.

The second term stands for the flow energy that comes from the ability of a fluid mass \( m = \rho \times V \) to do work \( W \) (N) generated by the earlier-mentioned pressure forces \( F = p \times A \). At pipe length \( L \), these forces create the work that can be described per unit mass as:

\[ W = FL = \frac{pAL}{\rho V} = \frac{p}{\rho} \]  \hspace{1cm} (3.13)

**Kinetic energy**

Finally, the third term in the equations represents the *kinetic energy* generated by the mass/volume motion.

By plugging 3.12 into 3.6, it becomes:

\[ Z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = Z_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \pm \Delta E \]  \hspace{1cm} (3.14)

**Bernoulli Equation**

In this form, the energy equation is known as the *Bernoulli Equation*. The equation parameters are shown in Figure 3.5. The following terminology is in common use:

- **Elevation head**: \( Z_{(1,2)} \)
- **Pressure head**: \( p_{(1,2)}/\rho g \)
- **Piezometric head**: \( H_{(1,2)} = Z_{(1,2)} + p_{(1,2)}/\rho g \)
- **Velocity head**: \( v^2_{(1,2)}/2g \)
- **Energy head**: \( E_{(1,2)} = H_{(1,2)} + v^2_{(1,2)}/2g \)

The pressure- and velocity-heads are expressed in mwc, which gives a good visual impression while talking about ‘high-’ or ‘low’ pressures or energies. The elevation-, piezometric- and energy heads are compared to a reference or ‘zero’ level. Any level can be taken as a reference; it is commonly the mean sea level suggesting the units for \( Z, H \) and \( E \) in *metres above mean sea level* (msl). Alternatively, the street level can also be taken as a reference level.
To provide a link with the SI-units, the following is valid:

- 1 mwc of the pressure head corresponds to 9.81 kPa in SI-units, which for practical reasons is often rounded off to 10 kPa.
- 1 mwc of the potential energy corresponds to 9.81 \( \frac{g}{10} \) kJ in SI-units; for instance, this energy will be possessed by 1 m\(^3\) of the water volume elevated 1 m above the reference level.
- 1 mwc of the kinetic energy corresponds to 9.81 \( \frac{g}{10} \) kJ in SI-units; for instance, this energy will be possessed by 1 m\(^3\) of the water volume flowing at a velocity of 1 m/s.

In reservoirs with a surface level in contact with the atmosphere, pressure \( p \) equals the atmospheric pressure, hence \( p = p_{\text{atm}} \approx 0 \). Furthermore, the velocity throughout the reservoir volume can be neglected (\( v = 0 \text{ m/s} \)). As a result, both the energy- and piezometric-head will be positioned at the surface of the water. Hence, \( E_{\text{tot}} = H = Z \).

The lines that indicate the energy- and piezometric-head levels in consecutive cross-sections of a pipe are called the energy grade line and the hydraulic grade line, respectively.

The energy and hydraulic grade line are parallel for uniform flow conditions. Furthermore, the velocity head is in reality considerably smaller than the pressure head. For example, for a common pipe velocity of 1 m/s, \( \frac{v^2}{2g} \approx 0.05 \text{ mwc} \), while the pressure heads are often in the order of tens of metres of water column. Hence, the real difference between these two lines is, with a few exceptions, negligible and the hydraulic grade line is predominantly considered while solving practical
problems. Its position and slope indicate:
– the pressures existing in the pipe, and
– the flow direction.

The hydraulic grade line is generally not parallel to the slope of the pipe that normally varies from section to section. In hilly terrains, the energy level may even drop below the pipe invert causing negative pressure (below atmospheric), as Figure 3.6 shows.

**Hydraulic gradient**

The slope of the hydraulic grade line is called the *hydraulic gradient*, \( S = \Delta E/L = \Delta H/L \), where \( L \) (m) is the length of the pipe section. This parameter reflects the pipe conveyance (Figure 3.7).

*The flow rate in pipes under pressure is related to the hydraulic gradient and not to the slope of the pipe. More energy is needed for a pipe to convey more water, which is expressed in the higher value of the hydraulic gradient.***

**PROBLEM 3.4**

For the pipe bend in Problem 3.3 (Section 3.1.1), calculate the total energy- and piezometric head in the cross-section of the bend if it is located at \( Z = 158 \text{ msl} \). Express the result in msl, J and kWh.
Answer:
In Problem 3.3, the pressure indicated in the pipe bend was $p = 100$ kPa, while the velocity, calculated from the flow rate and the pipe diameter, was $v = 0.37$ m/s. The total energy can be determined from Equation 3.12:

$$E_{\text{tot}} = Z + \frac{p}{\rho g} + \frac{v^2}{2g} = 158 + \frac{100,000}{1000 \times 9.81} + \frac{0.37^2}{2 \times 9.81}$$

$$= 158 + 10.194 + 0.007 = 168.2 \text{ msl}$$

As can be seen, the impact of the kinetic energy is minimal and the difference between the total energy and the piezometric head can therefore be neglected. The same result in J and kWh is as follows:

$$E_{\text{tot}} = 168.2 \times 1000 \times 9.81 = 1,650,042 \text{ J} \approx 1650 \text{ kJ (or kWs)}$$

$$= \frac{1650}{3600} = 0.5 \text{ kWh}$$

For an unspecified volume, the above result represents a type of unit energy, expressed per m$^3$ of water. To remember the units conversion: 1 N = 1 kg $\times$ m/s$^2$ and 1 J = 1 N $\times$ m.

3.2 HYDRAULIC LOSSES

The energy loss $\Delta E$ from Equation 3.14 is generated by:
– friction between the water and the pipe wall,
– turbulence caused by obstructions of the flow.

These causes inflict the friction- and minor losses, respectively. Both can be expressed in the same format:

$$\Delta E = h_f + h_m = R_f Q^\pi + R_m Q^n$$

(3.15)

Pipe resistance

where $R_f$ stands for resistance of a pipe with diameter $D$, along its length $L$. The parameter $R_m$ can be characterised as a resistance at the pipe cross-section where obstruction occurs. Exponents $n_f$ and $n_m$ depend on the type of equation applied.

3.2.1 Friction losses

The most popular equations used for the determination of friction losses are:
1 the Darcy–Weisbach Equation,
2 the Hazen–Williams Equation,
3 the Manning Equation.
Following the format in Equation 3.15:

**Darcy–Weisbach**

\[
R_f = \frac{8\lambda L}{\pi^2 g D^5} = \frac{\lambda L}{12.1D^5}; \quad n_f = 2
\]  

\[(3.16)\]

**Hazen–Williams**

\[
R_f = \frac{10.68L}{C_{hw} D^{1.87}}, \quad n_f = 1.852
\]

\[(3.17)\]

**Manning**

\[
R_f = \frac{10.29N^2 L}{D^{16/3}}, \quad n_f = 2
\]

\[(3.18)\]

In all three cases, the friction loss \(h_f\) will be calculated in mwc for the flow \(Q\) expressed in m³/s and length \(L\) and diameter \(D\) expressed in m. The use of prescribed parameter units in Equations 3.16–3.18 is to be strictly obeyed as the constants will need to be readjusted depending on the alternative units used.

In the above equations, \(\lambda\), \(C_{hw}\) and \(N\) are experimentally-determined factors that describe the impact of the pipe wall roughness on the friction loss.

**The Darcy–Weisbach Equation**

**Colebrook–White Equation**

In the Darcy–Weisbach Equation, the friction factor \(\lambda\) (\(\lambda\)) (also labelled as \(f\) in some literature) can be calculated from the equation of Colebrook–White:

\[
\frac{1}{\sqrt{\lambda}} = -2\log \left[ \frac{2.51}{Re\sqrt{\lambda}} + \frac{k}{3.7D} \right]
\]

\[(3.19)\]

where \(k\) is the absolute roughness of the pipe wall (mm), \(D\) the inner diameter of the pipe (mm) and \(Re\) the Reynolds number (\(\lambda\)).

To avoid iterative calculation, Barr (1975) suggests the following acceptable approximation, which deviates from the results obtained by the Colebrook–White Equation for \(\pm 1\%\):

\[
\frac{1}{\sqrt{\lambda}} = -2\log \left[ \frac{5.1286}{Re^{0.89}} + \frac{k}{3.7D} \right]
\]

\[(3.20)\]

**Reynolds number**

The Reynolds number describes the flow regime. It can be calculated as:

\[
Re = \frac{vD}{\nu}
\]

\[(3.21)\]
Kinematic viscosity

where \( v \) (m\(^2\)/s) stands for the kinematic viscosity. This parameter depends on the water temperature and can be determined from the following equation:

\[
v = \frac{497 \times 10^{-6}}{(T + 42.5)^{1.5}}
\]  

(3.22)

for \( T \) expressed in \(^\circ\)C.

The flow is:
1. laminar, if \( Re < 2000 \),
2. critical (in transition), for \( Re \approx 2000–4000 \),
3. turbulent, if \( Re > 4000 \).

The turbulent flows are predominant in distribution networks under normal operation. For example, within a typical range for the following parameters: \( v = 0.5–1.5 \) m/s, \( D = 50–1500 \) mm and \( T = 10–20\)^\circ\)C, the Reynolds number calculated by using Equations 3.21 and 3.22 has a value of between 19,000 and 225,000.

If for any reason \( Re < 4000 \), Equations 3.19 and 3.20 are no longer valid. The friction factor for the laminar flow conditions is then calculated as:

\[
\lambda = \frac{64}{Re}
\]  

(3.23)

As it usually results from very low velocities, this flow regime is not favourable in any way.

Once \( Re \), \( k \) and \( D \) are known, the \( \lambda \)-factor can also be determined from the Moody diagram, shown in Figure 3.8. This diagram is in essence a graphic presentation of the Colebrook–White Equation.

In the turbulent flow regime, Moody diagram shows a family of curves for different \( k/D \) ratios. This zone is split in two by the dashed line.

**Transitional turbulence zone**

The first sub-zone is called the transitional turbulence zone, where the effect of the pipe roughness on the friction factor is limited compared to the impact of the Reynolds number (i.e. the viscosity).

**Rough turbulence zone**

The curves in the second sub-zone of the rough (developed) turbulence are nearly parallel, which clearly indicates the opposite situation where the Reynolds number has little influence on the friction factor. As a result, in this zone the Colebrook–White Equation can be simplified:

\[
\frac{1}{\sqrt{\lambda}} = -2 \log \left[ \frac{k}{3.7D} \right]
\]  

(3.24)

For typical values of \( v \), \( k \), \( D \) and \( T \), the flow rate in distribution pipes often drops within the rough turbulence zone.
Figure 3.8. Moody diagram.

Table 3.1. Absolute roughness (Wessex Water PLC, 1993).

<table>
<thead>
<tr>
<th>Pipe material</th>
<th>$k$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asbestos cement</td>
<td>0.015–0.03</td>
</tr>
<tr>
<td>Galvanised/coated cast iron</td>
<td>0.06–0.3</td>
</tr>
<tr>
<td>Uncoated cast iron</td>
<td>0.15–0.6</td>
</tr>
<tr>
<td>Ductile iron</td>
<td>0.03–0.06</td>
</tr>
<tr>
<td>Uncoated steel</td>
<td>0.015–0.06</td>
</tr>
<tr>
<td>Coated steel</td>
<td>0.03–0.15</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.06–1.5</td>
</tr>
<tr>
<td>Plastic, PVC, PE</td>
<td>0.02–0.05</td>
</tr>
<tr>
<td>Glass fibre</td>
<td>0.06</td>
</tr>
<tr>
<td>Brass, Copper</td>
<td>0.003</td>
</tr>
</tbody>
</table>

**Absolute roughness**

The absolute roughness is dependant upon the pipe material and age. The most commonly used values for pipes in good condition are given in Table 3.1.

With the impact of corrosion, the $k$-values can increase substantially. In extreme cases, severe corrosion will be taken into consideration by reducing the inner diameter.
The Hazen–Williams Equation

The Hazen–Williams Equation is an empirical equation widely used in practice. It is especially applicable for smooth pipes of medium and large diameters and pipes that are not attacked by corrosion (Bhave, 1991). The values of the Hazen–Williams constant, \( C_{hw} \), for selected pipe materials and diameters are shown in Table 3.2.

Bhave states that the values in Table 3.2 are experimentally determined for flow velocity of 0.9 m/s. A correction for the percentage given in Table 3.3 is therefore suggested in case the actual velocity differs significantly. For example, the value of \( C_{hw} = 120 \) increases twice for 3\% if the expected velocity is around a quarter of the reference value i.e. \( C_{hw} = 127 \) for \( v \) of, say, 0.22 m/s. On the other hand, for doubled velocity \( v = 1.8 \) m/s, \( C_{hw} = 116 \) i.e. 3\% less than the original value of 120. However, such corrections do not significantly influence the friction loss calculation, and are, except for extreme cases, rarely applied in practice.

Bhave also states that the Hazen–Williams Equation becomes less accurate for \( C_{hw} \)-values below 100.

The Manning Equation

Strickler Equation

The Manning Equation is another empirical equation used for the calculation of friction losses. In a slightly modified format, it also occurs in some literature under the name of Strickler. The usual range of the \( N \)-values (m\(^{-1/3}\)s) for typical pipe materials is given in Table 3.4.

Table 3.2. The Hazen–Williams factors (Bhave, 1991).

<table>
<thead>
<tr>
<th>Pipe material</th>
<th>( C_{hw} ) 75 mm</th>
<th>( C_{hw} ) 150 mm</th>
<th>( C_{hw} ) 300 mm</th>
<th>( C_{hw} ) 600 mm</th>
<th>( C_{hw} ) 1200 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncoated cast iron</td>
<td>121</td>
<td>125</td>
<td>130</td>
<td>132</td>
<td>134</td>
</tr>
<tr>
<td>Coated cast iron</td>
<td>129</td>
<td>133</td>
<td>138</td>
<td>140</td>
<td>141</td>
</tr>
<tr>
<td>Uncoated steel</td>
<td>142</td>
<td>145</td>
<td>147</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Coated steel</td>
<td>137</td>
<td>142</td>
<td>145</td>
<td>148</td>
<td>148</td>
</tr>
<tr>
<td>Galvanized iron</td>
<td>129</td>
<td>133</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Uncoated asbestos cement</td>
<td>142</td>
<td>145</td>
<td>147</td>
<td>150</td>
<td>—</td>
</tr>
<tr>
<td>Coated asbestos cement</td>
<td>147</td>
<td>149</td>
<td>150</td>
<td>152</td>
<td>—</td>
</tr>
<tr>
<td>Concrete, minimum/maximum values</td>
<td>69/129</td>
<td>79/133</td>
<td>84/138</td>
<td>90/140</td>
<td>95/141</td>
</tr>
<tr>
<td>Pre-stressed concrete</td>
<td>—</td>
<td>—</td>
<td>147</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>PVC, Brass, Copper, Lead</td>
<td>147</td>
<td>149</td>
<td>150</td>
<td>152</td>
<td>153</td>
</tr>
<tr>
<td>Wavy PVC</td>
<td>142</td>
<td>145</td>
<td>147</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Bitumen/cement lined</td>
<td>147</td>
<td>149</td>
<td>150</td>
<td>152</td>
<td>153</td>
</tr>
</tbody>
</table>

Table 3.3. Correction of the Hazen–Williams factors (Bhave, 1991).

<table>
<thead>
<tr>
<th>( C_{hw} )</th>
<th>( v &lt; 0.9 ) m/s</th>
<th>( v &gt; 0.9 ) m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>less than 100</td>
<td>+5%</td>
<td>−5%</td>
</tr>
<tr>
<td>100–130</td>
<td>+3%</td>
<td>−3%</td>
</tr>
<tr>
<td>130–140</td>
<td>+1%</td>
<td>−1%</td>
</tr>
<tr>
<td>greater than 140</td>
<td>−1%</td>
<td>+1%</td>
</tr>
</tbody>
</table>
The Manning Equation is more suitable for rough pipes where $N$ is greater than $0.015 \text{ m}^{-1/3}\text{s}$. It is frequently used for open channel flows rather than pressurised flows.

Comparison of the friction loss equations
The straightforward calculation of pipe resistance, being the main advantage of the Hazen–Williams and Manning equations, has lost its relevance as a result of developments in computer technology. The research also shows some limitations in the application of these equations compared to the Darcy–Weisbach Equation (Liou, 1998). Nevertheless, this is not necessarily a problem for engineering practice and the Hazen–Williams Equation in particular is still widely used in some parts of the world.

Figures 3.9 and 3.10 show the friction loss diagrams for a range of diameters and two roughness values calculated by each of the three equations. The flow in two pipes of different length, $L = 200$ and $2000$ m

![Figure 3.9](image)

**Figure 3.9.** Comparison of the friction loss equations: mid range diameters, $v = 1$ m/s, $L = 200$ m.

<table>
<thead>
<tr>
<th>Pipe material</th>
<th>$N$ (m$^{-1/3}\text{s}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVC, Brass, Lead, Copper, Glass fibre</td>
<td>0.008–0.011</td>
</tr>
<tr>
<td>Pre-stressed concrete</td>
<td>0.009–0.012</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.010–0.017</td>
</tr>
<tr>
<td>Welded steel</td>
<td>0.012–0.013</td>
</tr>
<tr>
<td>Coated cast iron</td>
<td>0.012–0.014</td>
</tr>
<tr>
<td>Uncoated cast iron</td>
<td>0.013–0.015</td>
</tr>
<tr>
<td>Galvanised iron</td>
<td>0.015–0.017</td>
</tr>
</tbody>
</table>

Table 3.4. The Manning factors (Bhave, 1991).
respectively, is determined for velocity \( v = 1 \text{ m/s} \). Thus in all cases, for \( D \text{ in m and } Q \text{ in m}^3/\text{s} \):

\[
Q = \sqrt[4]{\frac{2}{\pi}} \approx 0.7854 D^2
\]

The example shows little difference between the results obtained by three different equations. Nevertheless, the same roughness parameters have a different impact on the friction loss in the case of larger and longer pipes.

The difference in results becomes larger if the roughness values are not properly chosen. Figure 3.11 shows the friction loss calculated using the roughness values suggested for PVC in Tables 3.1, 3.2 and 3.4.

Hence, the choice of a proper roughness value is more relevant than the choice of the friction loss equation itself. Which of the values fits the best to the particular case can be confirmed only by field measurements.

In general, the friction loss will rise when there is:

1. an increase in pipe discharge,
2. an increase in pipe roughness,
3. an increase in pipe length,
4. a reduction in pipe diameter,
5. a decrease in water temperature.

In reality, the situations causing this to happen are:

– higher consumption or leakage,
– corrosion growth,
– network expansion.

Figure 3.10. Comparison of the friction loss equations: large diameters, \( v = 1 \text{ m/s} \), \( L = 2000 \text{ m} \).
The friction loss equations clearly point to the pipe diameter as the most sensitive parameter. The Darcy–Weisbach Equation shows that each halving of \( D \) (e.g. from 200 to 100 mm) increases the head-loss \( 2^{5} = 32 \) times! Moreover, the discharge variation will have a quadratic impact on the head-losses, while these grow linearly with the increase of the pipe length. The friction losses are less sensitive to the change of the roughness factor, particularly in smooth pipes (an example is shown in Table 3.5). Finally, the impact of water temperature variation on the head-losses is marginal.

**PROBLEM 3.5**

For pipe \( L = 450 \text{ m}, D = 300 \text{ mm} \) and flow rate of 120 l/s, calculate the friction loss by comparing the Darcy–Weisbach- \((k = 0.2 \text{ mm})\), Hazen–Williams- \((C_{hw} = 125)\) and Manning equations \((N = 0.01)\). The water temperature can be assumed at 10 °C.

If the demand grows at the exponential rate of 1.8% annually, what will be the friction loss in the same pipe after 15 years? The assumed value of an increased absolute roughness in this period equals \( k = 0.5 \text{ mm} \).
Answer:
For a flow $Q = 120$ l/s and a diameter of 300 mm, the velocity in the pipe:

$$v = \frac{4Q}{D^2\pi} = \frac{4 \times 0.12}{0.3^2 \times 3.14} = 1.70 \text{ m/s}$$

Based on the water temperature, the kinematic viscosity can be calculated from Equation 3.22:

$$\nu = \frac{497 \times 10^{-6}}{(T + 42.5)^{1.5}} = \frac{497 \times 10^{-6}}{(10 + 42.5)^{1.5}} = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$$

The Reynolds number then becomes:

$$Re = \frac{\nu D}{\nu} = \frac{1.70 \times 0.3}{1.31 \times 10^{-6}} = 3.9 \times 10^5$$

For the value of relative roughness $k/D = 0.2/300 = 0.00067$ and the calculated Reynolds number, the friction factor $\lambda$ can be determined from the Moody diagram in Figure 3.8 ($\lambda = 0.019$). Based on the value of the Reynolds number ($> 4000$), the flow regime is obviously turbulent. The same result can also be obtained by applying the Barr approximation. From Equation 3.20:

$$\lambda = 0.25 \log^2 \left[ \frac{5.1286}{Re^{0.89}} + \frac{k}{3.7D} \right]$$

$$= 0.25 \log^2 \left[ \frac{5.1286}{(3.9 \times 10^5)^{0.89}} + \frac{0.2}{3.7 \times 300} \right] = 0.019$$

Finally, the friction loss from the Darcy–Weisbach Equation is determined as:

$$h_f = \frac{8\lambda L}{\pi^2 g D^5} Q^2 = \frac{\lambda L}{12.1 D^5} Q^2 = \frac{0.019 \times 450}{12.1 \times 0.3^5} 0.12^2 = 4.18 \text{ mwc}$$

Applying the Hazen–Williams Equation with $C_{hw} = 125$, the friction loss becomes:

$$h_f = \frac{10.68L}{C_{hw}^{1.852} D^{4.87} Q^{1.852}} = \frac{10.68 \times 450}{125^{1.852} 0.3^{4.87} 0.12^{1.852}} = 4.37 \text{ mwc}$$

Introducing a correction for the $C_{hw}$ value of 3%, as suggested in Table 3.3 based on the velocity of 1.7 m/s (almost twice the value of 0.9 m/s),
yields a value of \(C_{hw}\), which is reduced to 121. Using the same formula, the friction loss then becomes \(h_f = 4.64\) mwc, which is 6% higher than the initial figure.

Finally, applying the Manning Equation with the friction factor \(N = 0.01\):

\[
h_f = \frac{10.29N^2LQ^2}{D^{16/3}} = \frac{10.29 \times 0.01^2 \times 450}{0.3^{16/3}} 0.12^2 = 4.10\ mwc
\]

With the annual growth rate of 1.8%, the demand after 15 years becomes:

\[
Q_{15} = 120\left(1 + \frac{1.8}{100}\right)^{15} = 156.82\ l/s
\]

which, with the increase of the \(k\)-value to 0.5 mm, yields the friction loss of 8.60 mwc by applying the Darcy–Weisbach Equation in the same way as shown above. The interim calculations give the following values of the parameters involved: \(v = 2.22\ m/s, Re = 5.1 \times 10^5\) and \(\lambda = 0.023\). The final result represents an increase of more than 100% compared to the original value of the friction loss (at the demand increase of approximately 30%).

**Self-study:**
Workshop problems A1.2.1–A1.2.3 (Appendix 1)
Spreadsheet lessons A5.1.2 and A5.1.3 (Appendix 5)

### 3.2.2 Minor losses

*Minor* (in various literature *local* or *turbulence*) losses are usually caused by installed valves, bends, elbows, reducers, etc. Although the effect of the disturbance is spread over a short distance, the minor losses are for the sake of simplicity attributed to a cross-section of the pipe. As a result, an instant drop in the hydraulic grade line will be registered at the place of obstruction (see Figure 3.12).

Factors \(R_m\) and \(n_m\) from Equation 3.15 are uniformly expressed as:

\[
R_m = \frac{8\xi}{\pi^2 gD^4} = \frac{\xi}{12.1D^4}; \quad n_m = 2
\]

*Minor loss coefficient* where \(\xi\) represents the *minor (local) loss coefficient*. This factor is usually determined by experiments. The values for most typical appendages are given in Appendix 3. A very detailed overview can be found in Idel’cik (1986).
The minor loss factors for various types of valves are normally supplied together with the device. The corresponding equation may vary slightly from 3.25, mostly in order to enable a diagram that is convenient for easy reading of the values. In the example shown in Figure 3.13, the minor loss of a butterfly valve is calculated in mwc as: \( h_m = 10Q^2/K_v^2 \), for \( Q \) in m³/h. The \( K_v \)-values can be determined from the diagram for different valve diameters and settings.

Substantial minor losses are measured in the following cases:
1. the flow velocity is high, and/or
2. there is a significant valve throttling in the system.

Such conditions commonly occur in pumping stations and in pipes of larger capacities where installed valves are regularly operated; given the magnitude of the head-loss, the term ‘minor’ loss may not be appropriate in those situations. Within the distribution network on a large scale, the minor losses are comparatively smaller than the friction losses. Their impact on overall head-loss is typically represented through adjustment of the roughness values (increased \( k \) and \( N \) or reduced \( C_{hv} \)). In such cases, \( \Delta H \approx h_f \) is an acceptable approximation and the hydraulic gradient then becomes:

\[
S = \frac{\Delta H}{L} \approx \frac{h_f}{L}
\]  

**Equivalent pipe lengths**

The other possibility of considering the minor losses is to introduce so-called *equivalent pipe lengths*. This approach is sometimes used for the design of indoor installations where the minor loss impact is simulated by assuming an increased pipe length (for example, up to 30–40%) from the most critical end point.

### 3.3 SINGLE PIPE CALCULATION

Summarised from the previous paragraph, the basic parameters involved in the head-loss calculation of a single pipe using the
Darcy–Weisbach Equation are:
1 length $L$,
2 diameter $D$,
3 absolute roughness $k$,
4 discharge $Q$,
5 piezometric head difference $\Delta H$ (i.e. the head-loss),
6 water temperature $T$. 

3.13. Example of minor loss diagram from valve operation.
The parameters derived from the above are:
7 velocity, \( v = f(Q, D) \),
8 hydraulic gradient, \( S = f(\Delta H, L) \),
9 kinematic viscosity, \( v = f(T) \),
10 Reynolds number, \( Re = f(v, D, v) \),
11 friction factor, \( \lambda = f(k, D, Re) \).

In practice, three of the six basic parameters are always included as an input:
– \( L \), influenced by the consumers’ location,
– \( k \), influenced by the pipe material and its overall condition,
– \( T \), influenced by the ambient temperature.

The other three, \( D, Q \) and \( \Delta H \), are parameters of major impact on pressures and flows in the system. Any of these parameters can be considered as the overall output of the calculation after setting the other two in addition to the three initial input parameters. The result obtained in such a way answers one of the three typical questions that appear in practice:
1 What is the available head-loss \( \Delta H \) (and consequently the pressure) in a pipe of diameter \( D \), when it conveys flow \( Q \)?
2 What is the flow \( Q \) that a pipe of diameter \( D \) can deliver if certain maximum head-loss \( \Delta H_{\text{max}} \) (i.e. the minimum pressure \( p_{\text{min}} \)) is to be maintained?
3 What is the optimal diameter \( D \) of a pipe that has to deliver the required flow \( Q \) at a certain maximum head-loss \( \Delta H_{\text{max}} \) (i.e. minimum pressure \( p_{\text{min}} \))?

The calculation procedure in each of these cases is explained below. The form of the Darcy–Weisbach Equation linked to kinetic energy is more suitable in this case:

\[
\Delta H \approx h_f = \frac{\lambda L}{12.1D^2} Q^2 = \frac{\lambda L v^2}{D^2 g}; \quad S = \frac{\lambda v^2}{D^2 g} \tag{3.27}
\]

3.3.1 Pipe pressure

The input data in this type of the problem are: \( L, D, k, Q \) or \( v \), and \( T \), which yield \( \Delta H \) (or \( S \)) as the result. The following procedure is to be applied:
1 For given \( Q \) and \( D \), find out the velocity, \( v = 4Q/(D^2\pi) \).
2 Calculate \( Re \) from Equation 3.21.
3 Based on the \( Re \) value, choose the appropriate friction loss equation, 3.20 or 3.23, and determine the \( \lambda \)-factor. Alternatively, use the Moody diagram for an appropriate \( k/D \) ratio.
4 Determine \( \Delta H \) (or \( S \)) by Equation 3.27.
The sample calculation has already been demonstrated in Problem 3.5. To be able to define the pressure head, $p/\rho g$, an additional input is necessary:

- the pipe elevation heads, $Z$, and
- known (fixed) piezometric head, $H$, at one side.

There are two possible final outputs for the calculation:

1. If the downstream (discharge) piezometric head is specified, suggesting the minimum pressure to be maintained, the final result will show the required head/pressure at the upstream side i.e. at the supply point.
2. If the upstream (supply) piezometric head is specified, the final result will show the available head/pressure at the downstream side i.e. at the discharge point.

**PROBLEM 3.6**

The distribution area is supplied through a transportation pipe $L = 750\text{ m}$, $D = 400\text{ mm}$ and $k = 0.3\text{ mm}$, with the average flow rate of $1260\text{ m}^3/\text{h}$. For this flow, the water pressure at the end of the pipe has to be maintained at a minimum $30\text{ mwc}$. What will be the required piezometric level and also the pressure on the upstream side in this situation? The average pipe elevation varies from $Z_2 = 51\text{ msl}$ at the downstream side to $Z_1 = 75\text{ msl}$ at the upstream side. It can be assumed that the water temperature is $10^\circ\text{C}$.

**Answer:**

For flow $Q = 1260\text{ m}^3/\text{h} = 350\text{ l/s}$ and the diameter of 400 mm:

$$v = \frac{4Q}{D^2\pi} = \frac{4 \times 0.35}{0.4^2 \times 3.14} = 2.79\text{ m/s}$$

For temperature $T = 10^\circ\text{C}$, the kinematic viscosity from Equation 3.22, $\nu = 1.31 \times 10^{-6}\text{ m}^2/\text{s}$. The Reynolds number takes the value of:

$$Re = \frac{\nu D}{\nu} = \frac{2.79 \times 0.4}{1.31 \times 10^{-6}} = 8.5 \times 10^5$$

and the friction factor $\lambda$ from Barr’s Equation equals:

$$\lambda = 0.25 \left[ \log_2 \left( \frac{5.1286}{Re^{0.89}} + \frac{k}{3.7D} \right) \right]$$

$$= 0.25 \left[ \log_2 \left( \frac{5.1286}{(8.5 \times 10^5)^{0.89}} + \frac{0.3}{3.7 \times 400} \right) \right] \approx 0.019$$
The friction loss from the Darcy–Weisbach Equation can be determined as:

\[
h_f = \frac{\lambda L}{12.1D^2}Q^2 = \frac{0.019 \times 750}{12.1 \times 0.4^2} \times 0.35^2 \approx 14 \text{ mwc}
\]

The downstream pipe elevation is given at \(Z_2 = 51\) msl. By adding the minimum required pressure of 30 mwc to it, the downstream piezometric head becomes \(H_2 = 51 + 30 = 81\) msl. On the upstream side, the piezometric head must be higher for the value of calculated friction loss, which produces a head of \(H_1 = 81 + 14 = 95\) msl. Finally, the pressure on the upstream side will be obtained by deducting the upstream pipe elevation from this head. Hence \(p_1/\rho g = 95 - 75 = 20\) mwc. Due to configuration of the terrain in this example, the upstream pressure is lower than the downstream one. For the calculated friction loss, the hydraulic gradient \(S = h_f/L = 14/750 \approx 0.019\).

### 3.3.2 Maximum pipe capacity

For determination of the maximum pipe capacity, the input data are: \(L, D, k, \Delta H\) (or \(S\)), and \(T\). The result is flow \(Q\).

Due to the fact that the \(\lambda\)-factor depends on the Reynolds number i.e. the flow velocity that is not known in advance, an iterative procedure is required here. The following steps have to be executed:

1. Assume the initial velocity (usually, \(v = 1\) m/s).
2. Calculate \(Re\) from Equation 3.21.
3. Based on the \(Re\) value, choose the appropriate friction loss equation, 3.20 or 3.23, and calculate the \(\lambda\)-factor. For selected \(Re\)- and \(k/D\) values, the Moody diagram can also be used as an alternative.
4. Calculate the velocity after re-writing Equation 3.27:

\[
\nu = \sqrt{\frac{2gDS}{\lambda}} \quad (3.28)
\]

If the values of the assumed and determined velocity differ substantially, steps 2–4 should be repeated by taking the calculated velocity as the new input. When a sufficient accuracy has been reached, usually after 2–3 iterations for flows in the transitional turbulence zone, the procedure is completed and the flow can be calculated from the final velocity. If the flow is in the rough turbulence zone, the velocity obtained in the first iteration will already be the final one, as the calculated friction factor will remain constant (being independent from the value of the Reynolds number).

If the Moody diagram is used, an alternative approach can be applied for determination of the friction factor. The calculation starts by assuming the rough turbulence regime:

1. Read the initial \(\lambda\) value from Figure 3.8 based on the \(k/D\) ratio (or calculate it by applying Equation 3.24).
2 Calculate the velocity by applying Equation 3.28.
3 Calculate $Re$ from Equation 3.21.

Check on the graph if the obtained Reynolds number corresponds to the assumed $\lambda$ and $k/D$. If not, read the new $\lambda$-value for the calculated Reynolds number and repeat steps 2 and 3. Once a sufficient accuracy for the $\lambda$-value has been reached, the velocity calculated from this value will be the final velocity.

Both approaches are valid for a wide range of input parameters. The first one is numerical, i.e. suitable for computer programming. The second one is simpler for manual calculations; it is shorter and avoids estimation of the velocity in the first iteration. However, this approach relies very much on accurate reading of the values from the Moody diagram.

PROBLEM 3.7
For the system from Problem 3.6 (Section 3.1.1), calculate the maximum capacity that can be conveyed if the pipe diameter is increased to $D = 500$ mm and the head-loss has been limited to 10 m per km of the pipe length. The roughness factor for the new pipe diameter can be assumed at $k = 0.1$ mm.

Answer:
Assume velocity $v = 1$ m/s. For the temperature $T = 10^\circ$C, the kinematic viscosity from Equation 3.22, $v = 1.31 \times 10^{-6}$ m$^2$/s. With diameter $D = 500$ mm, the Reynolds number takes the value of:

$$Re = \frac{vD}{\nu} = \frac{1 \times 0.5}{1.31 \times 10^{-6}} = 3.8 \times 10^5$$

and the friction factor $\lambda$ from Barr’s Equation equals:

$$\lambda = 0.25 \left\{ \log^2 \left[ \frac{5.1286}{Re^{0.89}} + \frac{k}{3.7D} \right] \right\}$$

$$= 0.25 \left\{ \log^2 \left[ \frac{5.1286}{(3.8 \times 10^5)^{0.89}} + \frac{0.1}{3.7 \times 500} \right] \right\} \approx 0.016$$

The new value of the velocity based on the maximum-allowed hydraulic gradient $S_{max} = 10/1000 = 0.01$ is calculated from Equation 3.28:

$$v = \sqrt{\frac{2gDS}{\lambda}} = \sqrt{\frac{2 \times 9.81 \times 0.5 \times 0.01}{0.016}} = 2.48 \text{ m/s}$$
The result differs substantially from the assumed velocity and the calculation should be repeated in the second iteration with this value as a new assumption. Hence:

\[ Re = \frac{2.48 \times 0.5}{1.31 \times 10^{-6}} = 9.5 \times 10^5 \]

and the friction factor \( \lambda \) equals:

\[ \lambda = 0.25 \log^2 \left( \frac{5.1286}{(9.5 \times 10^5)^{0.89}} + \frac{0.1}{3.7 \times 500} \right) \approx 0.015 \]

The new resulting velocity will be:

\[ v = \sqrt{\frac{2 \times 9.81 \times 0.5 \times 0.01}{0.015}} = 2.57 \text{ m/s} \]

which can be considered as a sufficiently accurate result, as any additional iteration that can be done is not going to change this value. Finally, the maximum flow that can be discharged at \( S = 0.01 \) equals:

\[ Q = v \frac{D^2 \pi}{4} = 2.57 \frac{0.5^2 \times 3.14}{4} \approx 0.5 \text{ m}^3/\text{s} \approx 1800 \text{ m}^3/\text{h} \]

In the alternative approach, the initial \( \lambda \) value assuming the rough turbulent zone can be read from the Moody diagram in Figure 3.8. For a value of \( k/D = 0.1/500 = 0.0002 \), it is approximately 0.014. The calculation from the rewritten Equation 3.24 gives:

\[ \lambda = 0.25 \log^2 \left( \frac{k}{3.7D} \right) = 0.25 \log^2 \left( \frac{0.1}{3.7 \times 500} \right) = 0.0137 \]

With this value:

\[ v = \sqrt{\frac{2gDS}{\lambda}} = \sqrt{\frac{2 \times 9.81 \times 0.5 \times 0.01}{0.137}} = 2.66 \text{ m/s} \]

and the Reynolds number then becomes:

\[ Re = \frac{2.66 \times 0.5}{1.31 \times 10^{-6}} = 1.0 \times 10^6 \]

which means that the new reading for \( \lambda \) is closer to the value of 0.015 \( (k/D = 0.0002) \). Repeated calculation of the velocity and the Reynolds number with this figure leads to a final result as in the first approach.
3.3.3 Optimal diameter

In the calculation of optimal diameters, the input data are: \( L, k, Q, \Delta H \) (or \( S \)), and \( T \). The result is diameter \( D \).

The iteration procedure is similar to the one in the previous case, with the additional step of calculating the input diameter based on the assumed velocity:

1. Assume the initial velocity (usually, \( v = 1 \text{ m/s} \)).
2. Calculate the diameter from the velocity/flow relation. \( D^2 = 4Q/(\pi v) \).
3. Calculate \( Re \) from Equation 3.21.
4. Based on the \( Re \) value, choose the appropriate friction loss equation, 3.20 or 3.23, and determine the \( \lambda \)-factor. For selected \( Re \)- and \( k/D \) values, the Moody diagram can also be used instead.
5. Calculate the velocity from Equation 3.28.

If the values of the assumed and determined velocity differ substantially, steps 2–5 should be repeated by taking the calculated velocity as the new input.

After a sufficient accuracy has been achieved, the calculated diameter can be rounded up to a first higher (manufactured) size.

This procedure normally requires more iterations than for the calculation of the maximum pipe capacity. The calculation of the diameter from an assumed velocity is needed as the proper diameter assumption is often difficult and an inaccurate guess of \( D \) accumulates more errors than in the case of the assumption of velocity. For those reasons, the second approach in Section 3.3.2 is not recommended in this case.

PROBLEM 3.8
In case the flow from the previous problem has to be doubled to \( Q = 3600 \text{ m}^3/\text{h} \), calculate the diameter that would be sufficient to convey it without increasing the hydraulic gradient. The other input parameters remain the same as in Problem 3.7 (Section 3.3.2).

Answer:
Assume velocity \( v = 1 \text{ m/s} \). Based on this velocity, the diameter \( D \):

\[
D = 2\sqrt[4]{\frac{Q}{\pi v}} = 2\times \sqrt[4]{\frac{1}{1\times3.14}} = 1.128 \text{ m}
\]

and the Reynolds number:

\[
Re = \frac{vD}{v} = \frac{1\times1.128}{1.31\times10^{-6}} = 8.6\times10^5
\]
The friction factor $\lambda$ from Barr’s Equation equals:

$$\lambda = 0.25 \left[ \log_2 \left( \frac{5.1286}{Re^{0.89}} + \frac{k}{3.7D} \right) \right]$$

and at $S_{\text{max}} = 0.01$ the velocity from Equation 3.28 becomes:

$$v = \sqrt{\frac{2gDS}{\lambda}} = \sqrt{\frac{2 \times 9.81 \times 1.128 \times 0.01}{0.0135}} = 4.04 \text{ m/s}$$

The result is substantially different than the assumed velocity and the calculation has to be continued with several more iterations. The results after applying the same procedure are shown in the following table:

<table>
<thead>
<tr>
<th>Iter.</th>
<th>$v_{\text{ass}}$ (m/s)</th>
<th>$D$ (mm)</th>
<th>$Re$ (-)</th>
<th>$\lambda$ (-)</th>
<th>$v_{\text{calc}}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.04</td>
<td>561</td>
<td>$1.7 \times 10^6$</td>
<td>0.0141</td>
<td>2.79</td>
</tr>
<tr>
<td>3</td>
<td>2.79</td>
<td>676</td>
<td>$1.4 \times 10^6$</td>
<td>0.0139</td>
<td>3.09</td>
</tr>
<tr>
<td>4</td>
<td>3.09</td>
<td>642</td>
<td>$1.5 \times 10^6$</td>
<td>0.0139</td>
<td>3.01</td>
</tr>
<tr>
<td>5</td>
<td>3.01</td>
<td>650</td>
<td>$1.5 \times 10^6$</td>
<td>0.0139</td>
<td>3.03</td>
</tr>
</tbody>
</table>

with the final value for the diameter of $D = 650$ mm. The manufactured size would be, say, $D = 700$ mm.

Self-study:
Workshop problem A1.2.8 (Appendix 1)
Spreadsheet lesson A5.1.5 (Appendix 5)

3.3.4 Pipe charts and tables

Straightforward determination of the required pressures, flows or diameters is possible by using the pipe charts or pipe tables. These are created by combining the Darcy–Weisbach and Colebrook–White Equations. Substituting $\lambda$ and $Re$ in Equation 3.19, by using Equations 3.28 and 3.21 respectively, yields the following equation:

$$v = -2\sqrt{2gDS} \log \left[ \frac{2.51v}{DV\sqrt{2gDS}} + \frac{k}{3.7D} \right]$$  \hspace{1cm} (3.29)$$

For a fixed $k$-value and the water temperature (i.e. the viscosity), the velocity is calculated for common ranges of $D$ and $S$. The values for $v$, $D$, $S$ and $Q$ are then plotted or sorted in a tabular form (see Appendix 4).
The chart in Figure 3.14 shows an example of a flow rate of 20 l/s (top axis) passing through a pipe of diameter $D = 200$ mm (bottom axis). From the intersection of the lines connecting these two values it emerges that the corresponding velocity (left axis) and hydraulic gradient (right axis) would be around 0.6 m/s and 2 m/km, respectively. The same flow rate in a pipe $D = 300$ mm yields much lower values: the velocity would be below 0.3 m/s and the gradient around 0.3 m/km.

It is important to note that the particular graph or table is valid for one single roughness value and one single water temperature. Although the variation of these parameters has a smaller effect on the friction loss than the variation of $D$, $v$ or $Q$, this limits the application of the tables and graphs if the values specifically for $k$ differ substantially from those used in the creation of the table/graph. As an example, Table 3.6 shows the difference in the calculation of hydraulic gradients for the range of values for $k$ and $T$.

In former times, the pipe charts and tables were widely used for hydraulic calculations. Since the development of PC-spreadsheet programmes, their relevance has somewhat diminished. Nevertheless, they
are a useful help in providing quick and straightforward estimates of pipe discharges for given design layouts.

PROBLEM 3.9
Using the pipe tables, determine the maximum discharge capacity for pipe $D = 800$ mm for the following roughness values: $k = 0.01, 0.5, 1$ and $5$ mm and the maximum-allowed hydraulic gradients of $S = 0.001, 0.005, 0.01$ and $0.02$, respectively. The water temperature can be assumed at $T = 10\, ^\circ C$.

Answer:
The following table shows the results read for pipe $D = 800$ mm from the tables in Appendix 4:

<table>
<thead>
<tr>
<th>Parameter $T$</th>
<th>$k = 0.01$ mm</th>
<th>$k = 0.1$ mm</th>
<th>$k = 1$ mm</th>
<th>$k = 5$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = 10, ^\circ C$</td>
<td>0.0044</td>
<td>0.0052</td>
<td>0.0082</td>
<td>0.0133</td>
</tr>
<tr>
<td>$T = 20, ^\circ C$</td>
<td>0.0042</td>
<td>0.0051</td>
<td>0.0081</td>
<td>0.0132</td>
</tr>
<tr>
<td>$T = 40, ^\circ C$</td>
<td>0.0040</td>
<td>0.0049</td>
<td>0.0081</td>
<td>0.0132</td>
</tr>
</tbody>
</table>

The results suggest the following two conclusions:
1. For fixed values of $S$, the discharge capacity is reduced by the increase of the roughness value. In other words, the pipes start to lose their conveying capacity as they get older, which is reflected in reality by the drop of demand and/or pressure.
2. The discharge at the fixed $k$-value will increase by allowing the higher hydraulic gradient. In other words, if more of a friction loss is allowed in the network, more water will be distributed but at higher operational costs (because of additional pumping).

3.3.5 Equivalent diameters

During planning of network extensions or renovations, the alternative of laying single pipe or pipes connected in parallel or series is sometimes compared. To provide a hydraulically equivalent system, the capacity and hydraulic gradient along the considered section should remain
unchanged in all options. Those pipes are then of equivalent diameters (see Figure 3.15).

Each pipe in the parallel arrangement creates the same friction loss, which is equal to the total loss at the section. The total capacity is the sum of the flows in all pipes. Hence, for \( n \) pipes it is possible to write:

\[
\Delta H_{\text{equ}} = \Delta H_1 = \Delta H_2 = \cdots = \Delta H_n
\]

\[
Q_{\text{equ}} = Q_1 + Q_2 + \cdots + Q_n
\]

Pipes in parallel are more frequently of the same diameter, allowing for easier maintenance and handling of irregular situations. Furthermore, they will often be laid in the same trench i.e. along the same route and can therefore be assumed to be of the same length in which case the slope of the hydraulic grade line for all pipes will be equal. Nevertheless, the equation \( S_{\text{equ}} = S_1 = S_2 \cdots = S_n \) is not always true as the pipes connected in parallel need not necessarily be of identical length.

For pipes in series, the basic hydraulic condition is that each pipe carries the same flow rate. The total energy loss is the sum of the losses in all pipes. If written for \( n \) pipes:

\[
\Delta H_{\text{equ}} = \Delta H_1 + \Delta H_2 + \cdots + \Delta H_n
\]

\[
Q_{\text{equ}} = Q_1 = Q_2 = \cdots = Q_n
\]

Equation \( S_{\text{equ}} = S_1 + S_2 + \cdots + S_n \), will not normally be true except in the hypothetical case of \( S_1 = S_2 = \cdots = S_n \).

Figure 3.15. Equivalent diameters.
The hydraulic calculation of the equivalent diameters further proceeds based on the principles of the single pipe calculation, as explained in Paragraph 3.3.

PROBLEM 3.10
A pipe $L = 550$ m, $D = 400$ mm, and $k = 1$ mm transports the flow of 170 l/s. By an extension of the system this capacity is expected to grow to 250 l/s. Two alternatives to solve this problem are considered:
1. To lay a parallel pipe of smaller diameter on the same route, or
2. To lay a parallel pipe of the same diameter on a separate route with a total length $L = 800$ m.

Using the hydraulic tables for water temperature $T = 10^\circ$C:

a. Determine the diameter of the pipe required to supply the surplus capacity of 80 l/s in the first alternative,
b. Determine the discharge of the second pipe $D = 400$ mm in the second alternative.

In both cases, the absolute roughness of the new pipes can be assumed to be $k = 0.1$ mm.

**Answers:**
In the hydraulic tables in Appendix 4 (for $T = 10^\circ$C), the diameter $D = 400$ mm conveys the flow $Q = 156.6$ l/s for the hydraulic gradient $S = 0.005$ and $Q = 171.7$ l/s for $S = 0.006$. Assuming linear interpolation (which introduces negligible error), the flow of 170 l/s will be conveyed at $S = 0.0059$, leading to a friction loss $h_f = S \times L = 0.0059 \times 550 = 3.25$ mwc. This value is to be maintained in the design of the new parallel pipe.

Laying the second pipe in the same trench (i.e. with the same length) should provide an additional flow of 80 l/s. From the hydraulic tables for $k = 0.1$ mm the following closest discharge values can be read:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$D = 250$ mm</th>
<th>$D = 300$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = 0.005$</td>
<td>57.2</td>
<td>92.5</td>
</tr>
<tr>
<td>$S = 0.006$</td>
<td>62.9</td>
<td>101.7</td>
</tr>
</tbody>
</table>

which suggests that the manufactured diameter of 300 mm is the final solution. The flow rate to be conveyed at $S = 0.0059$ would be $Q = 100.8$ l/s (after interpolation) leading to a total supply capacity of 270.8 l/s.

In the second case, the parallel pipe $D = 400$ mm follows an alternative route with a total length of $L = 800$ m. The value of the hydraulic gradient will be consequently reduced to $S = 3.25 / 800 = 0.0041$. The
hydraulic tables give the following readings closest to this value:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$D = 400$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S = 0.004$</td>
<td>175.6</td>
</tr>
<tr>
<td>$S = 0.005$</td>
<td>197.3</td>
</tr>
</tbody>
</table>

Despite the longer route, this pipe is sufficiently large to convey capacities far beyond the required 80 l/s. For $S = 0.0041$, discharge $Q = 177.8$ l/s and the total supplying capacity from both pipes equals 347.8 l/s. Hence, more water but at higher investment costs.

**Self-study:**
Workshop problems A1.2.9–A1.2.11 (Appendix 1)
Spreadsheet lessons A5.2.1a–A5.2.5 (Appendix 5)

### 3.4 SERIAL AND BRANCHED NETWORKS

Calculation of serial and branched networks is entirely based on the methods used for single pipes. The differences in hydraulic performance occur between the branched systems with one supply point and those that have more than one supply point.

#### 3.4.1 Supply at one point

With known nodal demands, the flows in all pipes can easily be determined by applying the Continuity Equation (Equation 3.5), starting from the end points of the system (Figure 3.16).

![Figure 3.16. Branched network with a single supply point.](image)
If the diameters of the pipes are also known, the head-loss calculation follows the procedure in Section 3.3.1, resulting in the hydraulic gradient $S$ for each pipe. In the next step the piezometric heads, and consequently the pressures, will be calculated for each node starting from the node assumed to have the minimum pressure. In this respect potentially critical nodes are those with either high elevation and/or nodes located faraway from the source. Adding or subtracting the head-losses for each pipe, depending on the flow direction, will determine all other heads including the required piezometric head at the supply point. Calculation of the piezometric heads in the opposite direction, starting from the known value at the source, is also possible; this shows the pressures in the system available for specified head at the supplying point.

In situations where pipe diameters have to be designed, the maximum-allowed hydraulic gradient must be included in the calculation input. The iterative procedure from Section 3.3.3 or the pipe charts/tables are required here, leading to actual values of the hydraulic gradient for each pipe based on the best available (manufactured) diameter. Finally, the pressures in the system will also be determined either by setting the minimum pressure criterion or the head available at the supply point.

3.4.2 Supply at several points

For more than one supply point, the contribution from each source may differ depending on its piezometric head and distribution of nodal demands in the system. In this case, flows in the pipes connecting the sources are not directly known from the Continuity Equation. These flows can change their rate and even reverse the direction based on the variation of nodal demands. Figures 3.17 and 3.18 show an example of anticipated demand increase in node one.
Except for the chosen source, fixed conditions are required for all other sources existing in the system: a head, discharge or the hydraulic gradient of the connecting pipe(s). For the remainder, the calculation proceeds in precisely the same manner as in the case of one supply point.

**PROBLEM 3.11**

For the following branched system, calculate the pipe flows and nodal pressures for surface level in the reservoir of $H = 50$ msl. Assume for all pipes $k = 0.1$ mm, and water temperature of $10^\circ$C.

![Branched network with two supply points](image)

<table>
<thead>
<tr>
<th>Nodes: ID</th>
<th>Pipes: $L$(m)/$D$(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>530/250</td>
</tr>
<tr>
<td>2</td>
<td>410/150</td>
</tr>
<tr>
<td>3</td>
<td>630/200</td>
</tr>
<tr>
<td>4</td>
<td>540/100</td>
</tr>
<tr>
<td>5</td>
<td>580/150</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Z$ (msl)</th>
<th>12</th>
<th>22</th>
<th>17</th>
<th>25</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ (l/s)</td>
<td>-75.6</td>
<td>10.4</td>
<td>22.1</td>
<td>10.2</td>
<td>18.5</td>
</tr>
</tbody>
</table>
Answer:
The total supply from the reservoir equals the sum of all nodal demands, which is 75.6 l/s. Applying the Continuity Equation in each node (Equation 3.5), both the flow rate and its direction can be determined; each pipe conveys the flow that is the sum of all downstream nodal demands. The pipe friction loss will be further calculated by the approach discussed in Problem 3.6 (Section 3.3.1). If the hydraulic tables from Appendix 4 are used, the friction loss will be calculated from interpolated hydraulic gradients at a given diameter and flow rate (for fixed $k$ and $T$). The results of the calculation applying the Darcy–Weisbach Equation are shown in the following table.

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$Re$ (-)</th>
<th>$\lambda$ (-)</th>
<th>$S$ (-)</th>
<th>$L$ (m)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>250</td>
<td>75.6</td>
<td>1.54</td>
<td>$2.9 \times 10^5$</td>
<td>0.018</td>
<td>0.0086</td>
<td>530</td>
<td>4.55</td>
</tr>
<tr>
<td>2–3</td>
<td>150</td>
<td>22.1</td>
<td>1.25</td>
<td>$1.4 \times 10^5$</td>
<td>0.020</td>
<td>0.0108</td>
<td>410</td>
<td>4.43</td>
</tr>
<tr>
<td>2–5</td>
<td>200</td>
<td>43.1</td>
<td>1.37</td>
<td>$2.1 \times 10^5$</td>
<td>0.19</td>
<td>0.0090</td>
<td>630</td>
<td>5.70</td>
</tr>
<tr>
<td>5–4</td>
<td>100</td>
<td>10.2</td>
<td>1.30</td>
<td>$9.9 \times 10^4$</td>
<td>0.022</td>
<td>0.0192</td>
<td>540</td>
<td>10.38</td>
</tr>
<tr>
<td>5–6</td>
<td>150</td>
<td>14.4</td>
<td>0.81</td>
<td>$9.4 \times 10^4$</td>
<td>0.021</td>
<td>0.0048</td>
<td>580</td>
<td>2.78</td>
</tr>
</tbody>
</table>

Finally, the pressure in each node is calculated by subtracting the friction losses starting from the reservoir surface level and further deducting the nodal elevation from the piezometric heads obtained in this way. The final results are shown in the following table and figure.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ (msl)</td>
<td>50</td>
<td>45.45</td>
<td>41.02</td>
<td>29.37</td>
<td>39.75</td>
<td>36.96</td>
</tr>
<tr>
<td>$p$ (mwc)</td>
<td>—</td>
<td>33.45</td>
<td>19.02</td>
<td>12.37</td>
<td>14.75</td>
<td>16.96</td>
</tr>
</tbody>
</table>

Nodes: $p$ (mwc)
Pipes: $Q$ (l/s)
The lowest pressure appears to be in node 4 (12.4 mwc) resulting from a relatively small diameter (causing large friction loss) of pipe 5–4.

Self-study:
Workshop problems A1.3.1–A1.3.5 (Appendix 1)
Spreadsheet lessons A5.3.1–A5.3.2 (Appendix 5)

3.5 LOOPED NETWORKS

Kirchoff’s Laws

The principles of calculation as applied to single pipes are not sufficient in case of looped networks. Instead, a system of equations is required which can be solved by numerical algorithm. This system of equations is based on the analogy with two electricity laws known in physics as Kirchoff’s Laws. Translated to water distribution networks, these laws state that:

1. The sum of all ingoing and outgoing flows in each node equals zero ($\Sigma Q_i = 0$).
2. The sum of all head-losses along pipes that compose a complete loop equals zero ($\Sigma \Delta H_i = 0$).

The first law is essentially the mass conservation law, resulting in the Continuity Equation that must be valid for each node in the system.

From the second law, it emerges that the hydraulic grade line along one loop is also continuous, just as the flow in any node is. The number of equations that can be formulated applying this law equals the number of loops. For example, in the simple network from Figure 3.19 in the clockwise direction, this yields:

$$(H_r - H_1) + (H_1 - H_2) + (H_2 - H_3) - (H_r - H_3) = 0$$

Figure 3.19. Looped network.
3.5.1 Hardy Cross methods

Two similar iterative procedures can be derived from Kirchoff’s Laws:
1. The method of balancing heads.
2. The method of balancing flows.

These methods, known in literature under the name of Hardy Cross (published in 1936 and developed further by Cornish in 1939), calculate the pipe flows and nodal piezometric heads in looped systems for a given input, which is:
- for pipes: length $L$, diameter $D$, absolute roughness $k$ and minor loss factor $\xi$,
- for nodes: nodal discharge $Q$ and elevation $Z$.

Pressure in at least one node has to be fixed, which will influence the pressure in the rest of the system. This is usually a supply point.

Successive calculation of the loops (nodes) is executed by following the following steps:

**Method of balancing heads**
1. Flows from an initial guess are assigned to each pipe. However, these must satisfy the Continuity Equation in all nodes.
2. Head-loss in each pipe is calculated starting from Equation 3.15.
3. The sum of the head-losses along each loop is checked.
4. If the head-loss sum at any loop is outside of the required accuracy range, $0 \pm \varepsilon_{\Delta H}$ mwc, the following flow correction has to be introduced for each pipe within that loop (total $n$ pipes):

   $\delta Q_j = - \frac{\sum_{j=1}^{n} \Delta H_j}{2\sum_{j=1}^{n} |\Delta H_j/Q_j|} \quad (3.30)$

5. Correction $\delta Q$ is applied throughout the loop taking consistent orientation: clockwise or anti-clockwise. This has implications for the value of the pipe flows, which will be negative if their direction counters the adopted orientation. The positive/negative sign of the correction should also be taken into account while adding it to the current pipe flow.
6. The iteration procedure is carried out for the new flows, $Q + \delta Q$, repeated in steps 2–5, until $\varepsilon_{\Delta H}$ is satisfied for all loops.
7. After the iteration of flows and head-losses is completed, the pressures in the nodes can be determined from the reference node with fixed pressure, taking into account the flow directions.

The calculation proceeds simultaneously for all loops in the network, with their corresponding corrections $\delta Q$ being applied in the same iteration. In case of the pipes shared between the two neighbouring loops, the sum of the two $\delta Q$-corrections should be applied. The flow continuity in the nodes
will not be affected in this case; assuming uniform orientation for both loops will reverse the sign of the composite $\delta Q$ in one of them.

If the system is supplied from more than one source, the number of unknowns increases. Dummy loops have to be created by connecting the sources with dummy pipes of fictitious $L$, $D$ and $k$, but with a fixed $\Delta H$ equal to the surface elevation difference between the connected reservoirs. This value has to be maintained throughout the entire iteration process.

**Method of balancing flows**

1. The estimated piezometric heads are initially assigned to each node in the system, except for the reference i.e. fixed pressure node(s). An arbitrary distribution is allowed in this case.
2. The piezometric head difference is determined for each pipe.
3. Flow in each pipe is determined starting from the head-loss Equation 3.15.
4. The Continuity Equation is checked in each node.
5. If the sum of flows in any node is out of the requested accuracy range, $0 \pm \epsilon_Q \text{ m}^3/\text{s}$, the following piezometric head correction has to be introduced in that node ($n$ is the number of pipes connected in the node):

$$
\delta H_i = \frac{2\sum_{i=1}^{n} \Delta Q_i}{\sum_{i=1}^{n} |\Delta Q_i/\Delta H_i|}
$$

(3.31)

6. The iteration procedure is continued with the new heads, $H + \delta H$, repeated in steps 2–5, until $\epsilon_Q$ is satisfied for all nodes.

Unlike in the method of balancing heads, faster convergence in the method of balancing flows is reached by applying the corrections consecutively. As a consequence, the flow continuity in some nodes will include the pipe flows calculated from the piezometric heads of the surrounding nodes from the same iteration.

The required calculation time for both methods is influenced by the size of the network. The balancing head method involves systems with a smaller number of equations, equal to the number of loops, which saves time while doing the calculation manually. The balancing flow method requires a larger system of equations, equal to the number of nodes. However, this method excludes the identification of loops, which is of some advantage for computer programming.

The layout and operation of the system may have an impact on convergence in both methods. In general, faster convergence is reached by the balancing head method.

The Hardy Cross methods were widely used in the pre-computer era. The first hydraulic modelling software in water distribution was also based on these methods. Several modifications have been introduced in the meantime. The balancing flow method was first developed for
computer applications while the balancing head method still remains a preferred approach for manual calculations of simple looped networks. Both methods are programmable in spreadsheet form, which helps in reducing the calculation time in such cases.

PROBLEM 3.12
To improve the conveyance of the system from Problem 3.11, nodes one and four as well as nodes three and six have been connected with pipes $D = 100 \text{ mm}$ and $L = 1200$ and $1040 \text{ m}$, respectively ($k = 0.1 \text{ mm}$ in both cases). Calculate the pipe flows and nodal pressures for such a system by applying the balancing head method.

Answer:
Two loops are created from the branched system after adding the new pipes. The calculation starts by distributing the pipe flows arbitrarily, but satisfying the Continuity Equation in each node. The next step is to calculate the friction losses in each loop, as the following tables show (negative values mean the reverse direction, from the right node to the left one):

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>250</td>
<td>50.6</td>
<td>1.03</td>
<td>2.12</td>
</tr>
<tr>
<td>2–5</td>
<td>200</td>
<td>20.2</td>
<td>0.64</td>
<td>1.36</td>
</tr>
<tr>
<td>5–4</td>
<td>100</td>
<td>-14.8</td>
<td>-1.88</td>
<td>-21.17</td>
</tr>
<tr>
<td>4–1</td>
<td>100</td>
<td>-25.0</td>
<td>-3.18</td>
<td>-129.89</td>
</tr>
</tbody>
</table>

The sum of all friction losses, which should be equal to 0 for correct flow rate values, is in this case $\sum h_f = -147.59 \text{ mwc}$ (selecting the clockwise direction). Thus, the correction of all pipe flows in Loop One will be required in the new iteration. From Equation 3.30 ($\Delta H = h_f$) this correction becomes $\delta Q = 10.96 \text{ l/s}$. 
In the case of Loop Two:

The sum of all friction losses in this case is $\Sigma h_f = -2.35$ mwc, which is closer to the final result but also requires another flow correction. After applying Equation 3.30, $\delta Q = 1.21$ l/s.

In the second iteration, the following results were achieved after applying the pipe flows $Q + \delta Q$:

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>250</td>
<td>61.56</td>
<td>1.25</td>
<td>3.07</td>
</tr>
<tr>
<td>2–5</td>
<td>200</td>
<td>29.95</td>
<td>0.95</td>
<td>2.85</td>
</tr>
<tr>
<td>5–4</td>
<td>100</td>
<td>3.84</td>
<td>0.49</td>
<td>1.66</td>
</tr>
<tr>
<td>4–1</td>
<td>100</td>
<td>14.04</td>
<td>1.79</td>
<td>42.53</td>
</tr>
</tbody>
</table>

$\Sigma h_f = -38.27$ mwc and therefore $\delta Q = 5.31$ l/s.

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5–2</td>
<td>200</td>
<td>-29.95</td>
<td>-0.95</td>
<td>-2.85</td>
</tr>
<tr>
<td>2–3</td>
<td>150</td>
<td>21.21</td>
<td>1.20</td>
<td>4.10</td>
</tr>
<tr>
<td>3–6</td>
<td>100</td>
<td>-0.89</td>
<td>-0.11</td>
<td>-0.23</td>
</tr>
<tr>
<td>6–5</td>
<td>150</td>
<td>-15.29</td>
<td>-0.87</td>
<td>-3.12</td>
</tr>
</tbody>
</table>

$\Sigma h_f = -2.10$ mwc and therefore $\delta Q = 1.40$ l/s.

The new flow in pipes 2–5 shared between the loops has been obtained by applying the correction $\delta Q$ of both loops, i.e. $20.20 + 10.96 - 1.21 = 29.95$ l/s. This pipe in Loop Two has a reversed order of nodes and therefore $Q_{5-2} = -20.20 + 1.21 - 10.96 = -29.95$ l/s. Hence, the corrected flow of the shared pipe maintains the same value in both loops, once with a positive and once with a negative sign.

In the rest of the calculations:

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>250</td>
<td>66.86</td>
<td>1.36</td>
<td>3.60</td>
</tr>
<tr>
<td>2–5</td>
<td>200</td>
<td>33.85</td>
<td>1.08</td>
<td>3.60</td>
</tr>
<tr>
<td>5–4</td>
<td>100</td>
<td>1.46</td>
<td>0.19</td>
<td>0.29</td>
</tr>
<tr>
<td>4–1</td>
<td>100</td>
<td>-8.74</td>
<td>-1.11</td>
<td>-17.17</td>
</tr>
</tbody>
</table>

$\Sigma h_f = -9.69$ mwc and therefore $\delta Q = 2.09$ l/s.
### Loop two – Iteration three

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5–2</td>
<td>200</td>
<td>-33.85</td>
<td>-1.08</td>
<td>-3.60</td>
</tr>
<tr>
<td>2–3</td>
<td>150</td>
<td>22.61</td>
<td>1.28</td>
<td>4.63</td>
</tr>
<tr>
<td>3–6</td>
<td>100</td>
<td>0.51</td>
<td>0.06</td>
<td>0.09</td>
</tr>
<tr>
<td>6–5</td>
<td>150</td>
<td>-13.89</td>
<td>-0.79</td>
<td>-2.60</td>
</tr>
</tbody>
</table>

$\Sigma h_f = -1.48$ mwc and therefore $\delta Q = 1.11$ l/s.

### Loop one – Iteration four

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
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<tr>
<td>1–2</td>
<td>250</td>
<td>68.95</td>
<td>1.40</td>
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<tr>
<td>2–5</td>
<td>200</td>
<td>34.83</td>
<td>1.11</td>
<td>3.80</td>
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<tr>
<td>5–4</td>
<td>100</td>
<td>3.55</td>
<td>0.45</td>
<td>1.43</td>
</tr>
<tr>
<td>4–1</td>
<td>100</td>
<td>-6.65</td>
<td>-0.85</td>
<td>-10.25</td>
</tr>
</tbody>
</table>

$\Sigma h_f = -1.20$ mwc and therefore $\delta Q = 0.29$ l/s.

### Loop two – Iteration four

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5–2</td>
<td>200</td>
<td>-34.83</td>
<td>-1.11</td>
<td>-3.80</td>
</tr>
<tr>
<td>2–3</td>
<td>150</td>
<td>23.72</td>
<td>1.34</td>
<td>5.07</td>
</tr>
<tr>
<td>3–6</td>
<td>100</td>
<td>1.62</td>
<td>0.21</td>
<td>0.66</td>
</tr>
<tr>
<td>6–5</td>
<td>150</td>
<td>-12.78</td>
<td>-0.72</td>
<td>-2.22</td>
</tr>
</tbody>
</table>

$\Sigma h_f = -0.29$ mwc and therefore $\delta Q = 0.16$ l/s.

### Loop one – Iteration five

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>250</td>
<td>69.24</td>
<td>1.41</td>
<td>3.85</td>
</tr>
<tr>
<td>2–5</td>
<td>200</td>
<td>34.96</td>
<td>1.11</td>
<td>3.82</td>
</tr>
<tr>
<td>5–4</td>
<td>100</td>
<td>3.84</td>
<td>0.49</td>
<td>1.65</td>
</tr>
<tr>
<td>4–1</td>
<td>100</td>
<td>-6.36</td>
<td>-0.81</td>
<td>-9.44</td>
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</table>

$\Sigma h_f = -0.12$ mwc and therefore $\delta Q = 0.03$ l/s.

### Loop two – Iteration five

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$D$ (mm)</th>
<th>$Q$ (l/s)</th>
<th>$v$ (m/s)</th>
<th>$h_f$ (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5–2</td>
<td>200</td>
<td>-34.96</td>
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<td>-3.82</td>
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<td>2–3</td>
<td>150</td>
<td>23.88</td>
<td>1.35</td>
<td>5.14</td>
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<td>0.78</td>
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<tr>
<td>6–5</td>
<td>150</td>
<td>-12.62</td>
<td>-0.71</td>
<td>-2.17</td>
</tr>
</tbody>
</table>

$\Sigma h_f = -0.08$ mwc and therefore $\delta Q = 0.04$ l/s.
Loop one – Iteration six

<table>
<thead>
<tr>
<th>Pipe</th>
<th>(D) (mm)</th>
<th>(Q) (l/s)</th>
<th>(v) (m/s)</th>
<th>(h_f) (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–2</td>
<td>250</td>
<td>69.26</td>
<td>1.41</td>
<td>3.85</td>
</tr>
<tr>
<td>2–5</td>
<td>200</td>
<td>34.94</td>
<td>1.11</td>
<td>3.82</td>
</tr>
<tr>
<td>5–4</td>
<td>100</td>
<td>3.86</td>
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<td>1.68</td>
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<tr>
<td>4–1</td>
<td>100</td>
<td>–6.34</td>
<td>–0.81</td>
<td>–9.36</td>
</tr>
</tbody>
</table>

\(\sum h_f = -0.02\) mwc and therefore \(\delta Q = 0.00\) l/s.

Loop two – Iteration six

<table>
<thead>
<tr>
<th>Pipe</th>
<th>(D) (mm)</th>
<th>(Q) (l/s)</th>
<th>(v) (m/s)</th>
<th>(h_f) (mwc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5–2</td>
<td>200</td>
<td>–34.94</td>
<td>–1.11</td>
<td>–3.82</td>
</tr>
<tr>
<td>2–3</td>
<td>150</td>
<td>23.92</td>
<td>1.35</td>
<td>5.15</td>
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<td>3–6</td>
<td>100</td>
<td>1.82</td>
<td>0.23</td>
<td>0.82</td>
</tr>
<tr>
<td>6–5</td>
<td>150</td>
<td>–12.58</td>
<td>–0.71</td>
<td>–2.16</td>
</tr>
</tbody>
</table>

\(\sum h_f = -0.01\) mwc and therefore \(\delta Q = 0.01\) l/s.

As the tables show, the method already provides fast convergence after the first two to three iterations and continuation of the calculations does not add much to the accuracy of the results while it takes time, specifically in the case of manual calculations.

Determination of the nodal pressures will be done in the same way as in the case of branched systems: starting from the supply point with a fixed piezometric head, or from the point with minimum pressure required in the system. In each pipe, the unknown piezometric head on one side is obtained either by adding the pipe friction loss to the known downstream piezometric head, or by subtracting the pipe friction loss from the known upstream piezometric head. The final results are shown in the following figure:
3.5.2 Linear theory

The Newton Raphson and the Linear Theory methods succeeded the Hardy-Cross methods, as the main approach for solving the non-linear network governing equations. The linear theory method was developed by Wood and Charles, (1972) and involves a remarkably simple linearization compared with the Newton Raphson method. When using the Darcy–Weisbach Equation:

\[ \Delta H = (R_f + R_m)|Q| = UQ \]  \hspace{1cm} (3.32)

where:

\[ U = \frac{(\lambda L + \xi D)|Q|}{12.1D^5} \]  \hspace{1cm} (3.33)

The absolute value of \( Q \) helps to distinguish between different flow directions (+/− sign).

The following can be written for node \( i \), assuming the inflow to the node has a negative sign (Figure 3.20):

\[ Q_i - \sum_{j=1}^{n} Q_{ij} = 0 \]  \hspace{1cm} (3.34)

Index \( n \) represents the total number of pipes connected to node \( i \), while \( Q_i \) is the nodal demand (outflow).

Equation 3.34 is satisfied in the iteration procedure with specified accuracy \( \varepsilon_n \) for each node. Thus:

\[ Q_i - \sum_{j=1}^{n} Q_{ij} = \pm \varepsilon_i = f(H_i) \]  \hspace{1cm} (3.35)
and after combining Equations 3.32 and 3.35:

$$\pm \varepsilon_i = f(H_i) = Q_i - \sum_{j=1}^{n} H_j U_{ij} + H_i \sum_{j=1}^{n} \frac{1}{U_{ij}}$$

(3.36)

The system of linear equations given in 3.36, equals the total number of nodes in the network. The solution of these linear equations can be achieved by any standard procedure (i.e. algorithms for inversion and decomposition). For example, one could apply the Newton Raphson method that would give the nodal head in the \((k + 1)\)th iteration as follows:

$$H_i^{(k+1)} = H_i^{(k)} - \frac{f(H_i^{(k)})}{f'(H_i^{(k)})}$$

(3.37)

Plugging 3.36 into 3.37 yields:

$$f(H_i^{(k)}) = Q_i - \sum_{j=1}^{n} \frac{H_j^{(k)}}{U_{ij}^{(k)}} + H_i^{(k)} \sum_{j=1}^{n} \frac{1}{U_{ij}^{(k)}}$$

(3.38)

$$f'(H_i^{(k)}) = \sum_{j=1}^{n} \frac{1}{U_{ij}^{(k)}}$$

(3.39)

$$H_i^{(k+1)} = H_i^{(k)} - \left[ \frac{Q_i - \sum_{j=1}^{n} H_j^{(k)} / U_{ij}^{(k)} + H_i^{(k)} \sum_{j=1}^{n} 1 / U_{ij}^{(k)}}{\sum_{j=1}^{n} 1 / U_{ij}^{(k)}} \right]$$

(3.40)

Finally, a factor \(\omega\) that takes values between one and two can be added to improve the convergence:

$$H_i^{(k+1)} = H_i^{(k)}(1 - \omega) + \left[ \frac{\sum_{j=1}^{n} H_j^{(k)} / U_{ij}^{(k)} - Q_i}{\sum_{j=1}^{n} 1 / U_{ij}^{(k)}} \right] \omega$$

(3.41)

**Successive Over-Relaxation Method**

This method, known as the Successive Over-Relaxation Method (SOR), was recommended by Radojković and Klem (1989). Equation 3.41 shows that the piezometric head in node \(i\) depends on:
- piezometric heads at the surrounding nodes \(j = 1 \text{ to } n\), and
- resistance \(U\) of the pipes that connect node \(i\) with the surrounding nodes.

The size of the equation matrix in this approach is \(m \times n_{\text{max}}\), where \(m\) is the total number of nodes and \(n_{\text{max}}\) is a specified maximum number of nodes connected to each node in the system.
The iteration procedure is executed separately for nodes and pipes and consists of internal and external cycles. The preparation steps are:

1. Setting the initial values of the flow in each pipe. This is usually based on the velocity of 1 m/s and a given pipe diameter.
2. Calculation of the $U$-values based on the initial flows.
3. Setting the initial values of the piezometric head in each node. As in the case of the Hardy Cross methods, at least one node must be chosen as a reference (fixed head) node, to allow determination of other nodal heads in the system. The initial head in all other nodes can be selected in relation to the fixed head value.

The iteration starts in the internal cycle, where the nodal heads are determined by Equation 3.41. The calculation for each node is repeated until $H^{(k+1)} - H^{(k)} < \varepsilon_H$ is satisfied for all nodes in the system.

Thereafter, the flow will be calculated as $Q = \Delta H / U$ for each pipe, in the $(l+1)^{th}$ iteration of the external cycle. If $Q^{(l+1)} - Q^{(l)} > \varepsilon_Q$ for any of the pipes, the internal cycle will have to be repeated using the new $U$-values calculated from the latest flow rates.

The iteration stops when the requested flow accuracy has been achieved for all pipes or the specified maximum number of iterations was reached.

No matter how complicated the method looks at first glance, it is convenient for computer programming. Used for manual calculations, it will require lots of time even for a network of very few pipes. A spreadsheet application can reduce this but is by no means an alternative to a full-scale computer programme. It can however serve as a useful tool for better understanding of the principle.

**Gradient method**

The most recently used method to solve the network analysis problem, is the Gradient method that was first introduced by Todini and Pilati (1987). The linear system of equations formulated by their approach is solved using a matrix calculation described by George and Liu (1981). Next to its robustness, the additional advantage of this method is that it can handle the change of the status of system components (pumps and valves) without changing the structure of the equation matrix. The basic steps of the calculation procedure can be found in Rossman (2000).

**Self-study:**
Spreadsheet lesson A5.4.3 (Appendix 5)

### 3.6 PRESSURE-RELATED DEMAND

It is a known fact that more water is consumed when there is a higher pressure in the system, resulting in higher water flows at the outlets.
In addition to this, the leakage levels are very much pressure-sensitive, as can be seen from the British experience shown in Figure 3.21.

All calculation procedures explained in the previous sections deal with pipe flows against the hydraulic gradients, with the pressures calculated afterwards. As the reference head (pressure) is set independently from the flows, some error is introduced by neglecting the relation between the demand and pressure in the system. The mathematical relation between these two is quadratic, which can be derived from the analogy with the discharge through an orifice (see Figure 3.22).

The water pressure at the orifice is assumed to be atmospheric and applying the Bernoulli Equation for the cross-section just before and just after the orifice leads to the equation:

\[ Q = CA\sqrt{2gh} \]  

(3.42)

where \( A \) is the surface area of the orifice and \( C \) is a factor (< 1) related to its shape. With free surface level in the reservoir, water depth \( h \) above the orifice can be compared with energy head difference \( (\frac{h}{H})^2 \), while the \( C \)-factor corresponds to the minor loss factor,
Neglecting the friction, Equation 3.42 actually has a format comparable to $h_m = R_mQ^2$ shown in Equation 3.15 and further elaborated by Equation 3.25. Finally, it shows that the residual pressure in water distribution systems is destroyed at the tap, i.e. has in essence the status of a minor loss ($h_m = p/\rho g$).

In reality, applying this logic creates two potential problems:

1. Demand-driven hydraulic calculations will require a correction of the nodal discharges according to the calculated pressure, which may significantly increase the calculation time leading to an unstable iteration procedure. In other words, an input parameter (demand) becomes dependent on an output parameter (pressure).

2. Resistance $R_m$ is, in the case of hundreds of nodes supplying thousands of consumers, virtually impossible to determine. In the best possible case, a general pressure-related diagram may be created from a series of field measurements.

From the Dutch experience, KIWA suggest a linear relation between the pressure and demand for calculations carried out for the assessment of distribution network reliability. The demand is considered independently of the pressure above a certain threshold, which is typically a pressure of 20–30 mwc (Figure 3.23).

Running hydraulic calculations for low-pressure conditions without taking the pressure-related demand into consideration might result in negative pressures in some nodes. This happens if for example the supply head is too low (see Figure 3.24) or the head-losses in the system are exceptionally high. Proper interpretation of the results is necessary in this case in order to avoid false conclusions about the system operation.

Applying the pressure-related demand mode in calculations causes a gradual reduction of the nodal discharges and the hydraulic gradient values, resulting in the slower drop of the reservoir levels, as Figure 3.25 shows.

![Figure 3.23. Pressure-related demand relation (KIWA, 1993).](image-url)
Reservoirs and pumps are constructed to maintain the energy levels needed for water to reach the discharge points. Factors that directly influence the position, capacity and operation of these components are:
- topographical conditions,
- location of supply and demand points,
- patterns of demand variation,
- conveyance capacity of the network.

### 3.7.1 System characteristics

**Pipe characteristics**

The conveyance capacity of a pipe with a known length, diameter, roughness factor and slope is described by the pipe characteristics diagram. This diagram shows the required heads at the upstream side of the pipe, which enable the supply of a range of flows while maintaining constant pressure at the pipe end. The total head required for flow $Q$, in particular, consists of a dynamic and static component (see Figure 3.26).

**Dynamic head**

The dynamic head covers the head-losses i.e. the pipe resistance:

$$H_{\text{dyn}} = \Delta E = \Delta H = h_t + h_m$$  \hspace{1cm} (3.43)
Static head

The static head is independent of the flow:

\[ H_{st} = \frac{p_{\text{end}}}{\rho g} \pm \Delta Z \]  

(3.44)

where \( p_{\text{end}} \) stands for the remaining pressure at the pipe end. In design problems where the required head is to be determined for the maximum flow expected in the pipe, the pressure at the end will be fixed at a critical value, i.e. \( p_{\text{end}} = p_{\text{min}} \). Maintaining the specified minimum pressure at any flow \( Q \leq Q_{\text{max}} \) will result in the least energy input required for water conveyance at given pipe characteristics.

\( \Delta Z \) in Figure 3.26 is related to the pipe slope and represents the elevation difference between the end- and supplying point of the pipe. Positive \( \Delta Z \) indicates the necessity of pumping while, if there is a negative value, the gravity may partly or entirely be involved in water conveyance. In the example from the figure, the static head \( H_{st} = 10 \text{ mwc} \) comprises the minimum downstream pressure head of 5 mwc and 5 m of the elevation difference between the pipe ends. Such a pipe could deliver a maximum flow of 500 m\(^3\)/h if the head at the supply point was raised up to 22.5 mwc.

The curve for a single (transportation) pipe will be drawn for fixed \( L, D \) and \( k \)-values, and a range of flows covering the demand variations on a maximum consumption day. The minor losses are usually ignored and the curve will be plotted using the results of the friction loss calculation for various flow rates, as explained in Section 3.3.1.

System characteristics

In cases when the diagram in Figure 3.26 represents a network of pipes, it will be called the system characteristics. A quadratic relation between the system heads and flows can be assumed in this case:

\[ H_{\text{dyn}} = R_{s}Q^{2} \]  

(3.45)
where $R_s$ is the system resistance determined from pressure measurements for various demand scenarios.

The static head of the system will have the same meaning as in Formula 3.44 except that the end in this context means the most critical point i.e. with the lowest expected pressure. That point can exist at the physical end of the system, far away from the source, but can also be within the system if located at a high elevation (thus, inflicting high $\Delta Z$).

The system/pipe curves change their shape as the head-loss varies resulting from modification of the pipe roughness, diameter or length. This can be a consequence of:

– pipe ageing, and/or
– system rehabilitation/extension.

The curve becomes steeper in all cases as the head-loss increases, which results in the reduction of the conveying capacity. The original capacity can be restored by laying new pipes of a larger diameter, or in a parallel arrangement. Alternatively, more energy, i.e. the higher head at the supplying side, will be needed in order to meet the demand. The example in Figure 3.27 shows how the pipe from Figure 3.26 requires the upstream head of nearly 30 mwc after it loses its initial conveyance due to ageing, in order to keep the same supply of 500 m$^3$/h. Maintaining the initial supply head of 22.5 mwc would otherwise cause a reduction in supply to 400 m$^3$/h.

Self-study:
Spreadsheet lessons A5.1.6 and A5.2.1b (Appendix 5)

3.7.2 Gravity systems

In the case of gravity systems, the entire energy needed for water flow is provided from the elevation difference $\Delta Z$. The pressure variation in the

![Figure 3.27. Capacity reduction of the system.](image-url)
system is influenced exclusively by the demand variation (see Figure 3.28). Hence:

$$\Delta Z = H_{\text{dyn}} + H_{\text{st}} = \Delta H + \frac{P_{\text{end}}}{\rho g}$$  \hspace{1cm} (3.46)$$

For known pressure at the end of the system, the maximum capacity can be determined from the system curve, as shown in Figure 3.29.

The figure shows that the lower demand overnight causes smaller head-loss and therefore the minimum pressure in the system will be higher than during the daytime when the demand and head-losses are higher. In theory, this has implications for the value of the static head that is changing with the variable minimum pressure. The static head used for design purposes is always fixed based on the minimum pressure that is to be maintained in the system during the maximum consumption hour.

When an area that is to be supplied from a single source starts to grow considerably, demand increases and longer pipe routes can lead to a pressure drop in the network affecting the newly-constructed areas. In
theory, this problem can be solved by enlarging the pipes and/or elevating the reservoir (Figures 3.30 and 3.31). Nonetheless, the latter is often impossible due to the fixed position of the source and additional head will probably have to be provided by pumping.

**Zero-line**

If the system is supplied from more than one side, the storage that is at the higher elevation will normally provide more water i.e. the coverage of the larger part of the distribution area. The intersection between the hydraulic grade lines shows the line of separation between the areas covered by different reservoirs, the so-called zero-line (Figure 3.32).

Hydraulic conditions in the vicinity of the zero-line are unfavourable:
- the pressure is lower than in other parts of the network,
- the flow velocities are also low, leading to water stagnation and potential water quality problems.
PROBLEM 3.13

For the gravity system shown in the figure, find the diameter of the pipe \( L = 2000 \text{ m} \) that can deliver a flow of 6000 m\(^3\)/h with a pressure of 40 mwc at the entrance of the city. Absolute roughness of the pipe can be assumed at \( k = 1 \text{ mm} \) and the water temperature equals 10\(^\circ\)C.

What will the increase in capacity of the system be if the pressure at the entrance of the city drops to 30 mwc?

Answers:

At the elevation of \( Z = 10 \text{ msl} \) and the pressure of \( p/\rho g = 40 \text{ mwc} \), the piezometric head at the entrance of the city becomes \( H = 50 \text{ msl} \). The surface level/piezometric head of the reservoir is 10 metres higher, and this difference can be utilised as friction loss. The hydraulic gradient then becomes \( S = h_i/L = 10/2000 = 0.005 \). From the hydraulic tables in Appendix 4, for \( k = 1 \text{ mm} \) and \( T = 10^\circ\)C:

<table>
<thead>
<tr>
<th>Diameter (D mm)</th>
<th>Flow (Q m(^3)/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>4772.0</td>
</tr>
<tr>
<td>1000</td>
<td>6292.6</td>
</tr>
</tbody>
</table>
If the pressure drops to $p/\rho g = 30$ mwc, the available friction loss increases to $h_t = 20$ mwc and therefore $S = 0.01$. From the same tables, for $D = 1000$ mm, the flow that can be supplied for the given hydraulic gradient increases to $Q = 8911.1$ m$^3$/h.

The same results can be obtained by the calculation procedures demonstrated in Problems 3.8 and 3.7 in Sections 3.3.3 and 3.3.2, respectively.

**Self-study:**
Workshop problem A1.5.1 (Appendix 1)
Spreadsheet lessons A5.5.1–A5.5.5 (Appendix 5)

3.7.3 **Pumped systems**

In pumped systems, the energy needed for water conveyance is obtained from the pump operation. This energy, generated by the pump impeller, is usually expressed as a head of water column (in mwc) and is called the **pumping head** (or **pump lift**), $h_p$. It represents the difference between the energy levels at the pump entrance i.e. at the suction pipe and at the pump exit, i.e. at the discharge (or pressure) pipe (Figure 3.33).

In the case of a single pump unit, the higher the pumping head $h_p$ is, the smaller the pumped flow $Q$ will be. For a combination of $Q - h_p$ values, the power $N$ (W) required to lift the water is calculated as:

$$N = \rho g Q h_p$$  \hspace{1cm} (3.47)

where $Q$ (m$^3$/s) is the **pump discharge**. The power to drive the pump will be higher, due to energy losses in the pump:

$$N_p = \frac{\rho g Q h_p}{\eta_p}$$  \hspace{1cm} (3.48)

where $\eta_p$ is the pump efficiency dependant on the pump model and working regime. Finally, the power required for the pump motor will be:

$$N_m = \frac{N_p}{\eta_m}$$  \hspace{1cm} (3.49)

where $\eta_m$ indicates the motor efficiency.

![Figure 3.33. Pumping head.](image-url)
**Pump characteristics**

The hydraulic performance of pumps is described by the *pump characteristics*. This diagram shows the relation between the pump discharge and delivered head (Figure 3.34).

For centrifugal pumps, a very good approximation of the pump curve is achieved by the following equation:

\[ h_p = aQ^2 + bQ + c \]  \hspace{1cm} (3.50)

where factors \(a\), \(b\) and \(c\) depend on the pump model and flow units. The alternative equation can also be used:

\[ h_p = c - aQ^b \]  \hspace{1cm} (3.51)

**Duty flow and head**

This equation allows for the pump curve definition with a single set of \(Q-h_p\) points (Rossman, 2000). These are known as the duty flow \((Q_d)\) and duty head \((H_d)\) and indicate the optimal operational regime of the pump, i.e. the one in which the maximum efficiency \(\eta_p\) will be achieved. As a convention, for exponent \(b = 2\):

\[ h_{p,(Q=0)} = c = \frac{4}{3}H_d; \quad Q_{(h_p=0)} = 2Q_d \Rightarrow a = \frac{H_d}{3Q_d} \]  \hspace{1cm} (3.52)

Pump manufacturers regularly supply pump characteristics diagrams for each model; a typical format showing a range of impeller diameters and efficiencies \(\eta_p\), is given in Figure 3.35.

Following the discussions in Section 3.7.1, the pumping head required at the supply side of the system to maintain certain minimum pressure at its end will be:

\[ h_{p,req} = H_{dyne} + H_{st} = \Delta H + \frac{P_{min}}{\rho g} \pm \Delta Z \]  \hspace{1cm} (3.53)

This required head is normally higher during daytime than overnight, resulting from higher demand i.e. higher head-losses. Operating the
same pump (curve) over 24 hours is therefore unfavourable as it results in the opposite effect: low heads during the daytime and high heads overnight (Figure 3.36). Apart from that, using a single pump in a pumping station is unjustified for reasons of low reliability, high-energy consumption/low efficiency and problematic maintenance. In practice, several pumps are commonly combined in one pumping station.

**Working point**

Flows and pressures that can be delivered by pump operation are determined from the system and pump characteristics. The intersection of these two curves, the *working point*, indicates the required pumping head that provides the flow and the static head, as shown on the graph in Figure 3.37.
As in the case of the gravity supply discussed in the previous section, the pressure variation in the distribution area has implications for the value of the static head in this case as well. For design purposes, however, the working point obtained from the system characteristics plotted at the lowest static head (i.e. the lowest pressure required in the system) will be used to determine the maximum pump capacity.

The maximum pumping capacity may vary in time. Ageing of the pump impeller, pipe corrosion, increase of leakage, etc. will cause reduction of the maximum flow that can be delivered by the pump while maintaining the same static head (Figure 3.38).

Decisions on the number and size of pumps in a pumping station are made with the general intention of keeping the pressure variations in the system at the lowest acceptable level in order to minimise the required pumping energy. For this reason, several pumps connected to the same delivery main can be installed in parallel. Their operation will be represented by a composite pump curve, which is obtained by adding the single pump discharges at the same pumping head. Hence, for $n$ pumps:

\[ h_p = h_1 = h_2 = h_3 = \cdots = h_n \]
\[ Q_p = Q_1 + Q_2 + Q_3 + \cdots + Q_n \]
Figure 3.39 shows the operation of two equal pumping units in parallel arrangement. The system should preferably operate at any point along the curve A₁-A₂-B₁-B₂, between the minimum and maximum flow.

The shaded area on the figure indicates excessive pumping, which is unavoidable when fixed speed pumps are used. A properly-selected combination of pump units should reduce this area. This is often achieved by installing pumps of different capacities; the example in Figure 3.40 shows the combination of three equal units with one stronger pump.

Introducing variable speed pumps can completely eliminate the excessive head. The flow variation is in this case met by adjusting the impeller rotation, keeping the discharge pressure constant (Figure 3.41).

The pump characteristics diagram will consist of a family of curves for various pump frequencies, $n$ (rpm). The relation between the various pumping heads and flows of any two curves is proportional to the frequencies in the following way:

$$\frac{Q_2}{Q_1} = \frac{n_2}{n_1}; \quad \frac{h_{p,2}}{h_{p,1}} = \left(\frac{n_2}{n_1}\right)^2$$

$$\tag{3.54}$$
Nevertheless, one variable speed unit alone can hardly cover the entire range of flows and several units in parallel will therefore be used. Variable speed pumps can also be combined with fixed speed pumps, controlling only the peak flows (Figure 3.42).

In case of large pressure variations in the system, the pumps have to be installed in a serial arrangement. The total head is in this case equal to the sum of heads for each pump. Figure 3.43 shows the curves for two equal units:

\[
\begin{align*}
    h_p &= h_1 + h_2 + h_3 + \cdots + h_n \\
    Q_p &= Q_1 = Q_2 = Q_3 = \cdots = Q_n
\end{align*}
\]

In addition to the discussion at the end of Section 3.7.2, pumping from more than one supply point will cause similar problems as with gravity systems in areas where the water from different sources is mixed. However, modifying the pump regimes can shift the zones of minimum pressure, as shown in Figure 3.44.
PROBLEM 3.14

For the pumped system shown in the following figure, determine the required pumping head to deliver flow \( Q = 4000 \text{ m}^3/\text{h} \) through pipe \( L = 1200 \text{ m} \) and \( D = 800 \text{ mm} \), while maintaining the pressure of 50 mwc at the entrance of the city. Absolute roughness of the pipe can be assumed at \( k = 0.5 \text{ mm} \) and the water temperature equals 10\(^\circ\)C.

Find the equation of the pump curve using Equations 3.51 and 3.52 assuming the earlier operation happens at maximum pump efficiency. What will be the pressure at the entrance of the city when there is a demand growth of 25%?
Answers:

At the entrance of the city, elevation $Z = 40$ msl. With a required pressure of $p/\rho g = 50$ mwc, the piezometric head becomes $H = 90$ msl. As the surface level/piezometric head of the reservoir is set at 20 metres, the total static head $H_{st} = 90 - 20 = 70$ mwc. The losses between the reservoir and the pump can be ignored.

For the following parameters: $Q = 4000$ m$^3$/h, $L = 1200$ m, $D = 800$ mm, $k = 0.5$ mm and $T = 10^\circ$C, the pipe friction loss will be calculated as follows:

$$v = \frac{4Q}{D^2\pi} = \frac{4 \times 4000}{0.8^2 \times 3.14 \times 3600} = 2.21 \text{ m/s}$$

For temperature $T = 10^\circ$C, the kinematic viscosity from Equation 3.22, $\nu = 1.31 \times 10^{-6}$ m$^2$/s. The Reynolds number takes the value of:

$$Re = \frac{vD}{\nu} = \frac{2.21 \times 0.8}{1.31 \times 10^{-6}} = 1.4 \times 10^6$$

and the friction factor $\lambda$ from Barr’s Equation equals:

$$\lambda = 0.25 \log^2 \left[ \frac{5.1286}{Re^{0.89}} + \frac{k}{3.7D} \right]$$

$$= 0.25 \log^2 \left[ \frac{5.1286}{(1.4 \times 10^6)^{0.89}} + \frac{0.5}{3.7 \times 800} \right] \approx 0.018$$

The friction loss from the Darcy–Weisbach Equation can be determined as:

$$h_f = \frac{\lambda L}{12.1D^5Q^2} = \frac{0.018 \times 1200}{12.1 \times 0.8^5} \times 1.11^2 \approx 7 \text{ mwc}$$

The total required pumping head is therefore $h_p = H_{st} + H_{\text{dyn}} = 70 + 7 = 77$ mwc. Given the maximum pumping efficiency, this is also the duty head at the duty flow of 4000 m$^3$/h and in Equation 3.51:

$$a = \frac{1}{3} \frac{H_d}{Q_d^2} = \frac{77}{3 \times 4000^2} = 1.604 \times 10^{-6} \text{ and}$$

$$c = \frac{4}{3} H_d = \frac{4}{3} \times 77 = 102.67$$
Hence, the pumping curve can be approximated with the following equation (exponent $b = 2$):

$$h_p = c - aQ^b = 103 - 1.6 \times 10^{-6} Q^2$$

If demand grows by 25% i.e. to 5000 m$^3$/h, the pumping head that can be provided will be:

$$h_p = 103 - 1.6 \times 10^{-6} 5000^2 \approx 63 \text{ mwc}$$

The friction loss calculated in the same way as above is going to increase to approximately 11 mwc, leading to a residual pressure at the entrance to the city of $p/\rho g = 20 + 63 - 11 - 40 \approx 32 \text{ mwc}$. 

**Self-study:**
Workshop problem A5.2 (Appendix 1)
Spreadsheet lessons A5.6.1–A5.6.5 (Appendix 5)

### 3.7.4 Combined systems

Consumers in combined systems are partly supplied by gravity and partly by pumping. Three basic concepts can be distinguished:

1. The water is pumped from a reservoir into the distribution area (tank-pump-network).
2. The water is pumped to a reservoir and thereafter supplied by gravity (pump-tank-network).
3. Pump and reservoir are at the opposite sides of the distribution area (pump-network-tank).

**Tank-pump-network**

This scheme is suitable for mild terrains where a favourable location for the reservoir is difficult to find, either due to insufficiently-high elevations or because of a large distance from the distribution area.

Essentially this is the same concept as that of direct pumping, except that the required pumping head can be reduced on account of the elevation difference in the system. Hence:

$$h_p + \Delta Z = H_{dyn} + H_{st} = \Delta H + \frac{P_{end}}{\rho g}$$ (3.55)

Both the dynamic and static head are supplied partly by gravity and partly by pumping, depending on the elevation difference and the pressure at the end of the system (Figures 3.45 and 3.46).
Pumping stations need not necessarily to be located at the supply point. When positioned within the system, they are commonly called *booster stations*. Such a layout is attractive if high pressures are to be avoided (Figure 3.47).

**Pump-tank-network**

This scheme is typical for hilly terrains. When pumps deliver water to the reservoir, the static head will only comprise the elevation head $\Delta Z$, which equals the elevation difference between the surface levels in the two reservoirs (Figure 3.48). Thus:

$$h_p = H_{\text{dyn}} + H_{\text{st}} = \Delta H + \Delta Z$$

(3.54)

The advantages of this scheme are:
- stable operation of the pumping station,
- a buffer supply capacity in case of pump failure.

A similar hydraulic pattern is valid if water towers are put into the system (Figure 3.49). However, their predominant role is to maintain stable...
Figure 3.47. Booster stations.

Figure 3.48. Gravity supply supported by pumping.

Figure 3.49. Pump operation in combination with water tower.
operation of the pumps, rather than to provide buffer- or large balancing volumes.

While supplying tanks, the pumps often operate automatically, based on monitoring of water levels in the reservoirs. Pump throttling may be required in order to adjust the flow. The effects on the system characteristics are shown in Figure 3.50.

![Figure 3.50. Effects of pump throttling on system characteristics.](image)

![Figure 3.51. Counter tank: daytime flows.](image)

![Figure 3.52. Counter tank: night time flows.](image)
Steady Flows in Pressurised Networks

**Pump-network-tank**
This scheme is predominantly applied for distribution networks located in valleys. During the maximum supply conditions, both the pump and reservoir will cover part of the distribution area (Figure 3.51). If the only source of supply is close to the pumping station, that one will also be used to refill the volume of the tank. This is usually done overnight when the demand in the area is low (Figure 3.52).

**Counter tank**
Tank operating in this way functions as a kind of *counter tank* to the one at the source. Depending on its size and elevation, it can balance the demand variation in the system, partly or completely. In the second case, the pumping station operates at constant (average) capacity ($Q_{\text{pump}} = Q_{\text{average}}$).

*Self-study:*
Workshop problems A1.5.3–A1.5.5 (Appendix 1)
Spreadsheet lessons A5.7.1–A5.7.4 (Appendix 5)